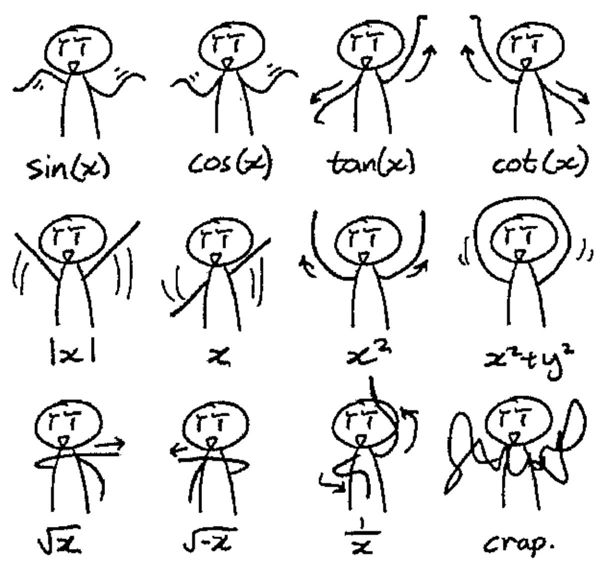
Name: KEY!

Hour: _____

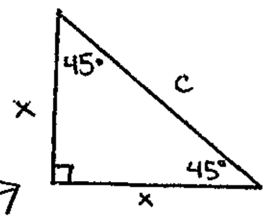
Unit M: Trigonometry

Geometry 2nd Semester

Beautiful Dance Moves



Lesson 13-5: Special Right Triangles



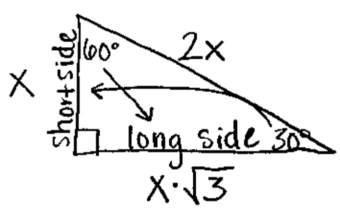
$c^2 = x^2 + x^2$
$c^2 = 2x^2$
$c = \sqrt{2x^2}$
$c = x\sqrt{2}$

Vocabulary

Isosceles Right Triangle Theorem: in an isoscles right triangle, if the legs are x, then the hypotenuse is $x\cdot\sqrt{2}$.

30-60-90 Triangle Theorem: in a 30°, 60°, 90° triangle, if the shorter leg is x, then the longer side is $x\cdot\sqrt{3}$ & the hypotenuse is $2\cdot x$.

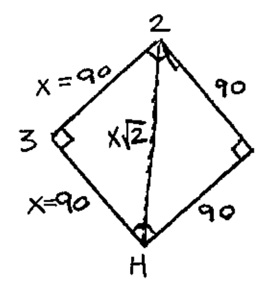
Picture:



Practice

1. How far is it from home to second base?

$x = 90$
Home to 2nd is the hypotenuse,
so $x\sqrt{2}$ ← exact ↙ rounded
$= \boxed{90\sqrt{2}}$ or 127.3



Isosc. Rt. Δ

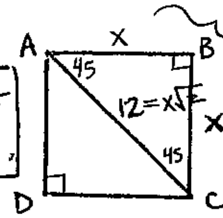
2. What is the exact value for the length of a side of the square?

Hypotenuse is $\frac{x\sqrt{2}}{\sqrt{2}} = \frac{12}{\sqrt{2}}$ * rationalize the denom.!

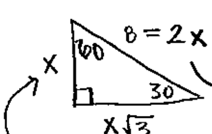
$x = \frac{12}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{12\sqrt{2}}{2} = \boxed{6\sqrt{2}}$ ← exact

or 8.49 ← rounded

ΔABC is isosc. rt.



3. Find YZ and XZ.

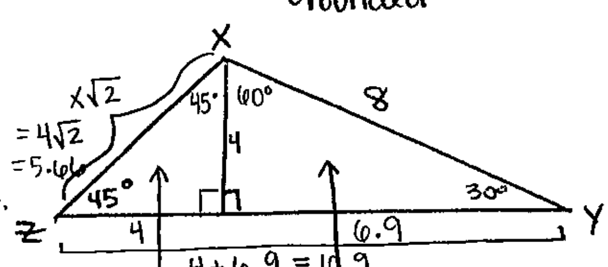


$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$

$x = 4$

$4\sqrt{3} \approx 6.9$

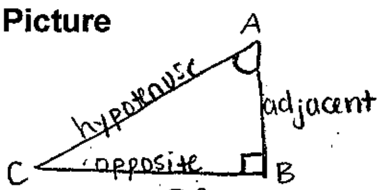
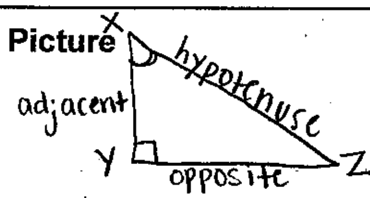
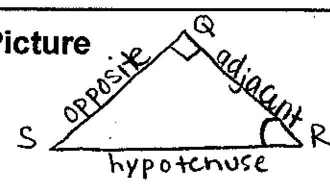
$\boxed{\begin{array}{l} YZ = 10.9 \\ XZ = 5.66 \end{array}}$



Lesson 13-6 & 13-7: SOH-CAH-TOA

NOTE: Calculators must be in "degree mode".

Vocabulary

Sine	Cosine	Tangent
Abbreviation sin	**Abbreviation** cos	**Abbreviation** tan
Symbols SOH	**Symbols** CAH	**Symbols** TOA
Words $\sin = \frac{\text{opposite}}{\text{hypotenuse}}$	**Words** $\cos = \frac{\text{adjacent}}{\text{hypotenuse}}$	**Words** $\tan = \frac{\text{opposite}}{\text{adjacent}}$
Picture  $\sin A = \frac{BC}{AC}$	**Picture**  $\cos X = \frac{XY}{XZ}$	**Picture**  $\tan R = \frac{QS}{QR}$

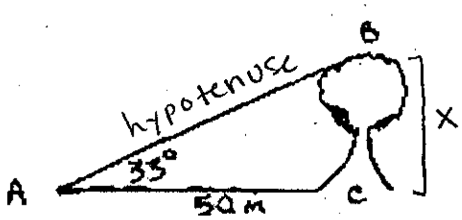
use the inverse of each, $\sin^{-1}$, $\cos^{-1}$, and $\tan^{-1}$ to find missing angle measures

Practice

1. Find the tan26°.

$\tan 26° \approx 0.49$

2. At a location 50m from the base of a tree, the angle of elevation of the tree top is 33°. Determine the height of the tree to the nearest meter.



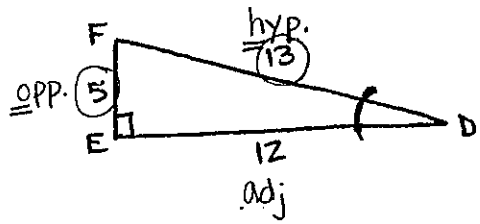
Since we have opposite & adjacent, we will use tan.

$$\tan 33° = \frac{x}{50}$$

$$50 \cdot .649 = \frac{x}{50} \cdot 50$$

$$\boxed{32.45 \approx x}$$

3. In the right triangle, find m∠D. SOH-CAH-TOA



SOH

$\sin D = \frac{5}{13}$

$D = \sin^{-1}\left(\frac{5}{13}\right)$

$\boxed{D = 22.6^\circ}$

4. Find each ratio for the triangle at the right.

a. SOH sin A = $5/13$

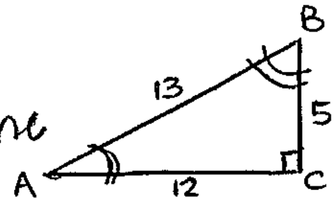
b. CAH cos A = $12/13$

c. SOH sin B = $12/13$

d. CAH cos B = $5/13$

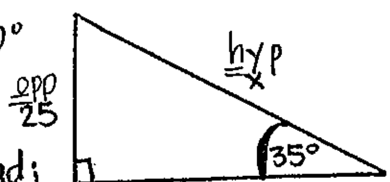
same

same



5. Find the missing side length, x, for the triangle below.

Steps:
1) Choose non-90° angle
2) Label sides w/ opp/hyp/adj
3) Decide: SOH-CAH-TOA
4) Set-up & solve equation



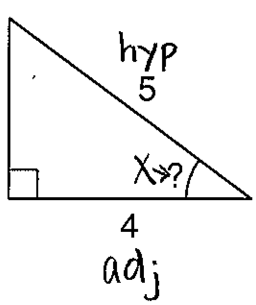
SOH

$x \cdot \sin 35^\circ = \frac{25}{\cancel{x}} \cdot x$

$\frac{x \cdot \cancel{\sin 35^\circ}}{\cancel{\sin 35}} = \frac{25}{\sin 35}$

$\boxed{x = 43.6}$

6. Find the missing angle below.



CAH

$\cos x = \frac{4}{5}$

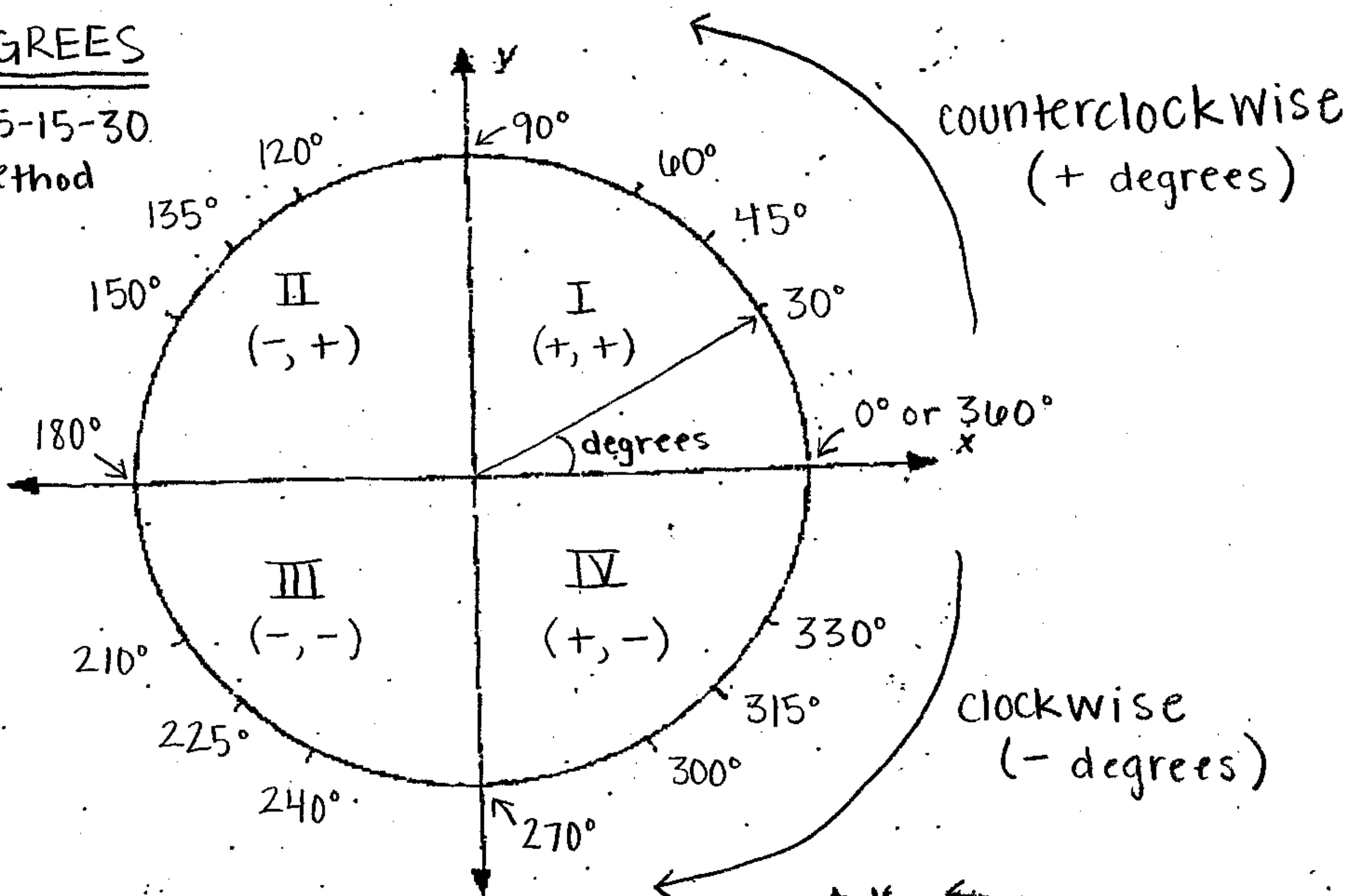
$x = \cos^{-1}\left(\frac{4}{5}\right)$

$\boxed{x = 36.9^\circ}$

Unit Circle Notes

DEGREES

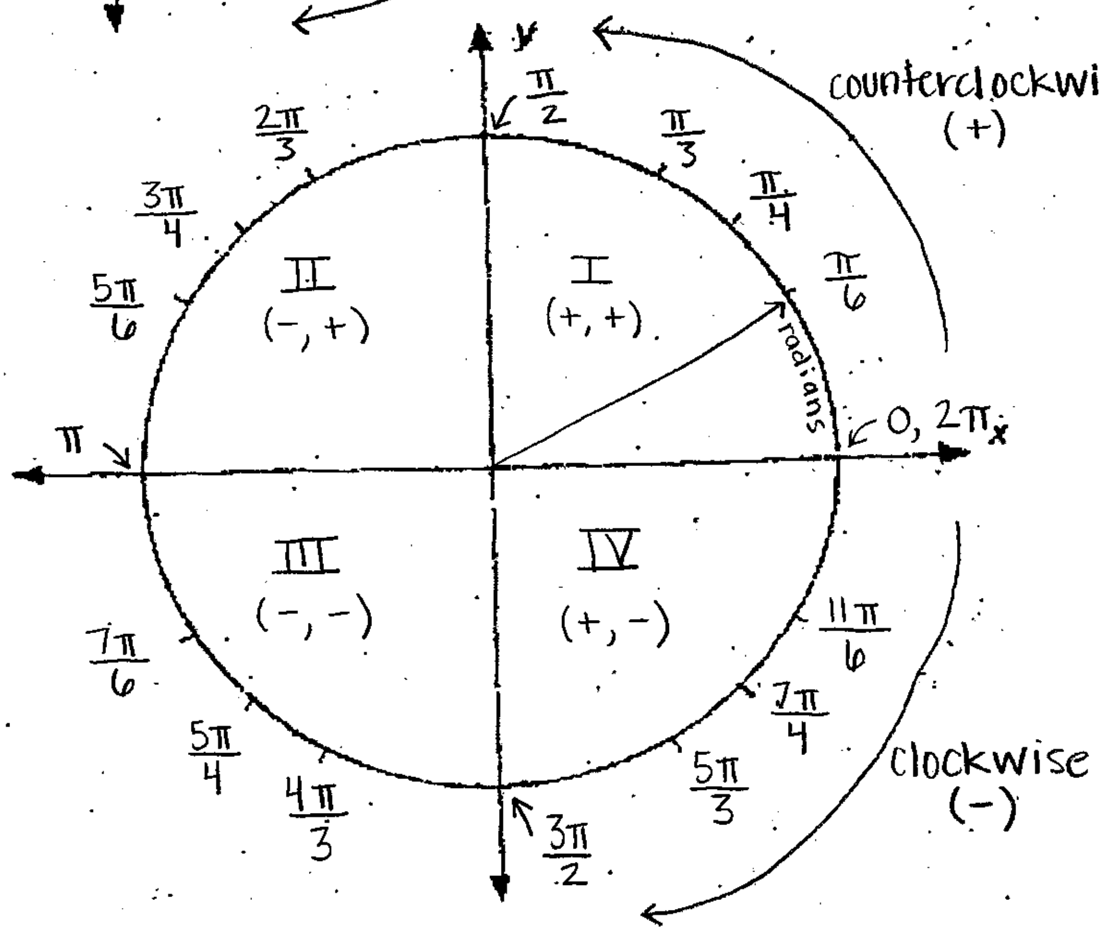
30-15-15-30 method



RADIANS

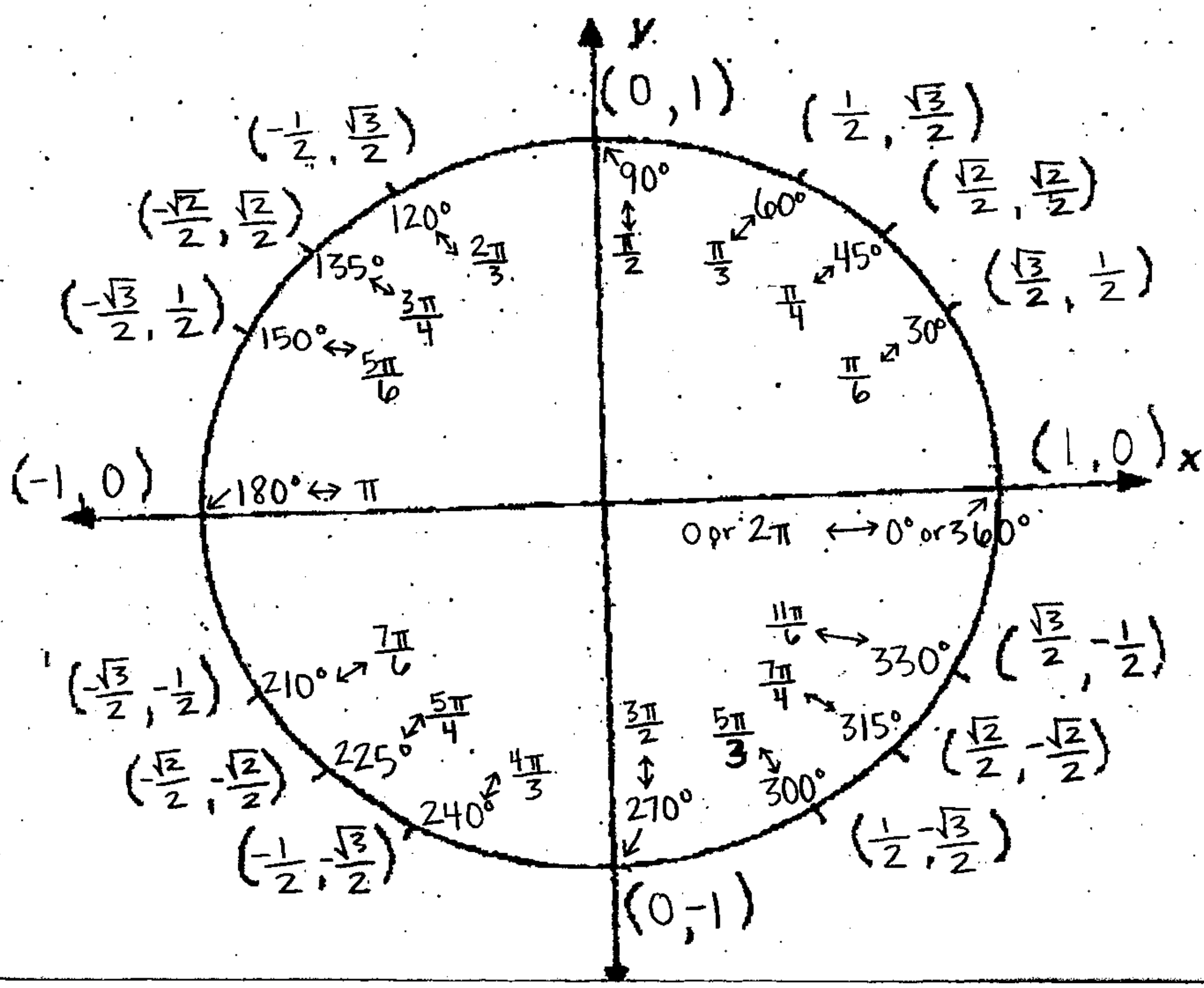
$2\pi, \pi, \frac{\pi}{2}, \frac{\pi}{4}, \frac{\pi}{6}$

method



Unit Circle Notes (Continued...)

DEGREES, RADIANS, (cos, sin)



Practice * look @ unit circle!

1. $\sin 30°$

$1/2$

2. $\cos 210°$

$-\sqrt{3}/2$

3. $\cos \frac{3\pi}{4}$

$-\sqrt{2}/2$

4. $\sin \frac{11\pi}{6}$

$-1/2$

5. $\tan 30° = \frac{\sin 30}{\cos 30}$

$$\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

} rationalize the denom! illegal!

$$\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$$

6. $\tan \frac{\pi}{3} = \frac{\sin \frac{\pi}{3}}{\cos \frac{\pi}{3}}$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{\cancel{2}} \cdot \frac{\cancel{2}}{1} = \frac{\sqrt{3}}{1} = \boxed{\sqrt{3}}$$

7. $\sin(-90°)$

-1

8. $\cos(-150°)$

$-1/2$

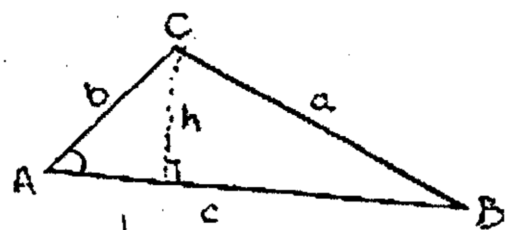
Law of Sines & Law of Cosines

Vocabulary

Law of Sines: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ in a triangle with ASA, AAS, & SSA.

Law of Cosines: $a^2 = b^2 + c^2 - 2bc \cdot \cos A$ & $b^2 = a^2 + c^2 - 2ac \cdot \cos B$, & $c^2 = a^2 + b^2 - 2ab \cdot \cos C$ in a triangle with SAS or SSS.

How in the world did they figure that out?!? Check this out...



1) $\sin A = \frac{h}{b}$ and $\sin B = \frac{h}{a}$

2) Solve for h in the above equations.

$b \cdot \sin A = \frac{h}{\cancel{b}} \cdot \cancel{b}$ & $a \cdot \sin B = \frac{h}{\cancel{a}} \cdot \cancel{a}$

$b \cdot \sin A = h$

$a \cdot \sin B = h$

3) Since both equations are equal to h, we can set them equal to each other:

$$b \cdot \sin A = a \cdot \sin B$$

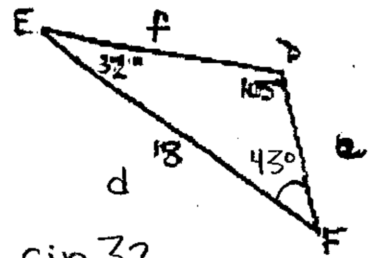
4) Using the above equations, divide each side by a and b.

$$\frac{\cancel{b} \cdot \sin A}{a \cdot \cancel{b}} = \frac{\cancel{a} \cdot \sin B}{\cancel{a} \cdot b}$$

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

Practice

1. For the triangle given, find:



a. ∠F.

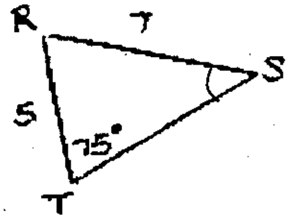
$$180-(32+105)=43°$$

b. The length of side "e".

$$\frac{\sin D}{d}=\frac{\sin E}{e}, \quad \frac{\sin 105}{18}=\frac{\sin 32}{e}$$

$$\boxed{e=9.88}$$

2. Find the measure of ∠S.



$$\frac{\sin S}{s}=\frac{\sin T}{t}$$

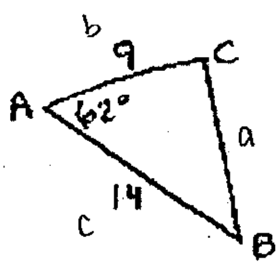
$$\frac{\sin S}{5}=\frac{\sin 75}{7}$$

$$\sin S=.6899$$

$$S=\sin^{-1}(.6899)$$

$$\boxed{S\approx 43.6°}$$

3. Find BC.



$$a^2=b^2+c^2-2bc\cdot\cos A$$

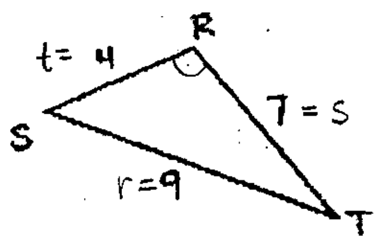
$$a^2=9^2+14^2-2\cdot9\cdot14\cdot\cos 62°$$

$$a^2=277-252\cos 62$$

$$\sqrt{a^2}=\sqrt{158.7}$$

$$\boxed{a\approx 12.6}$$

4. Find m∠R.



$$r^2=s^2+t^2-2st\cdot\cos R$$

$$9^2=7^2+4^2-2\cdot7\cdot4\cdot\cos R$$

$$81=65-56\cos R$$

$$-65 \qquad -65$$

$$\frac{16}{-56}=\frac{-56\cos R}{-56}$$

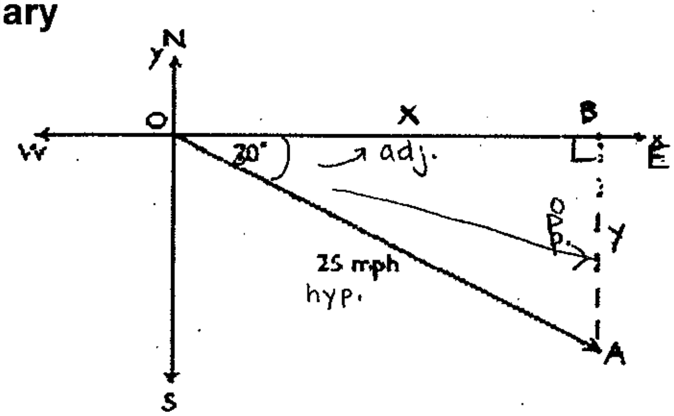
$$-.2857=\cos R$$

$$\cos^{-1}(-.2857)=R$$

$$\boxed{R\approx 106.6°}$$

Lesson 13-8: Vectors & Area

Vocabulary



Vector A is 30° clockwise from positive x axis

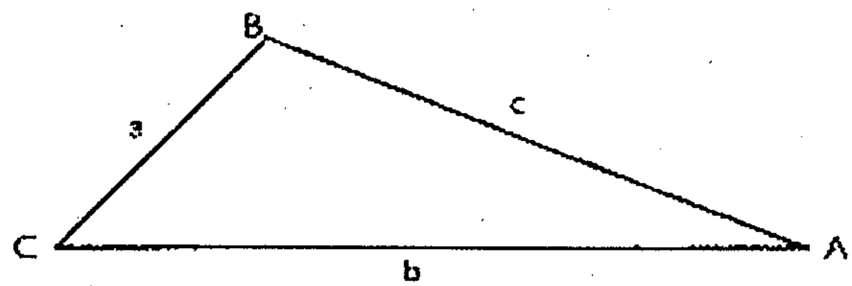
Vector A is 30° south of east

Vector: has a terminal point & initial point

Velocity: is $\overrightarrow{OA}$ (speed)

Magnitude: is 25 miles

SAS Triangle Area Formula: In $\triangle ABC$,
area $(\triangle ABC) = \frac{1}{2} \cdot a \cdot b \cdot \sin C$



Practice

1. Find the eastern component of x in the above picture.

$$25 \cdot \cos 30° = \frac{x}{\cancel{25}} \cdot \cancel{25}$$

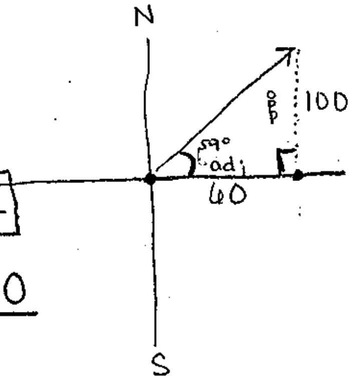
$$\boxed{21.65 = x}$$

2. Find the southern component of ~~y~~ in the above picture.

$$25 \cdot \sin 30° = \frac{y}{\cancel{25}} \cdot \cancel{25}$$

$$\boxed{12.5 = y}$$

3. An airplane has to fly to a location 60km east and 100km north of its present location.



a. In what direction should it fly? TOA

$\tan\theta = \frac{100}{60}$ $\rightarrow$ $\theta = \tan^{-1}\left(\frac{100}{60}\right)$

$\boxed{\theta = 59° \text{ north of east}}$

b. How far will it travel?

$a^2 + b^2 = c^2$

$60^2 + 100^2 = c^2$

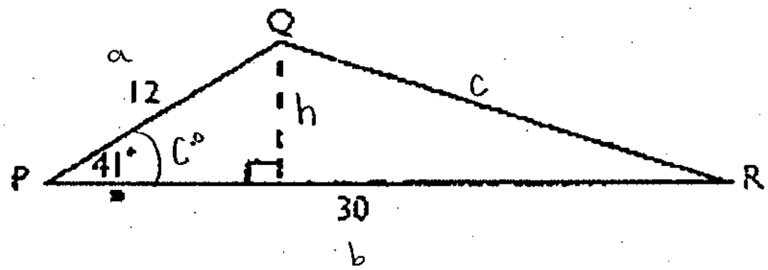
$\boxed{116.6 = c}$

OR

$\frac{\sin 59}{100} = \frac{\sin 90}{x}$

$\boxed{x = 116.6}$

4. Find the area of ΔPQR.



Option # 1:
find h, use $A = \frac{1}{2} \cdot b \cdot h$

$\frac{\sin 41}{h} = \frac{\sin 90}{12}$

$h \approx 7.87$

$\text{Area} = \frac{1}{2} \cdot 7.87 \cdot 30$

$= 118.05$

Option #2:
Use SAS Δ Area Formula

$\text{Area}(\Delta ABC) = \frac{1}{2} \cdot a \cdot b \cdot \sin C$

$= \frac{1}{2} \cdot 12 \cdot 30 \cdot \sin 41°$

$= 118.09$

same answer
(or really close ☺)