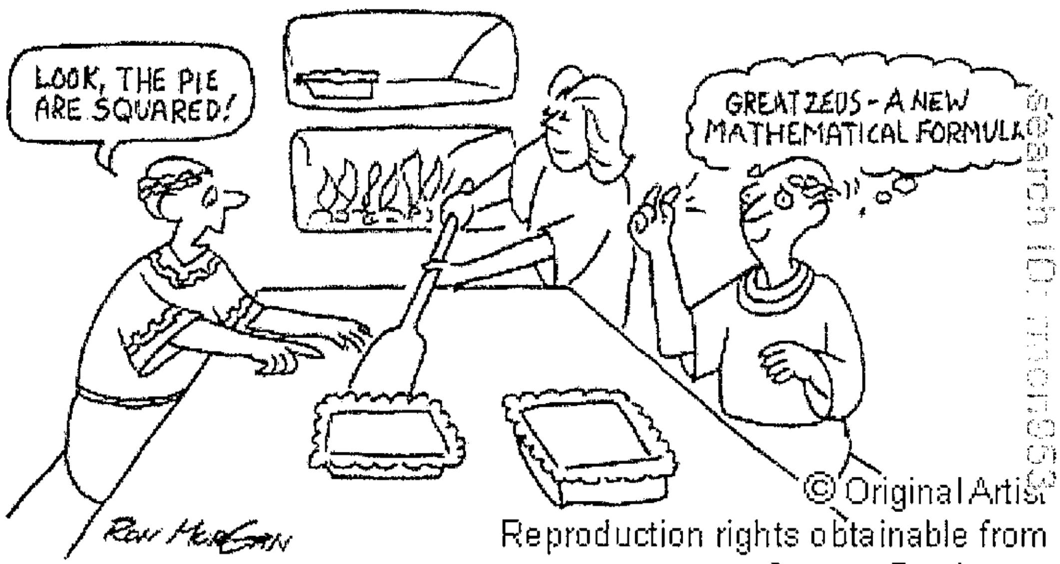
Name: KEY

Hour: ______

Unit K: Similarity

Geometry 2nd Semester



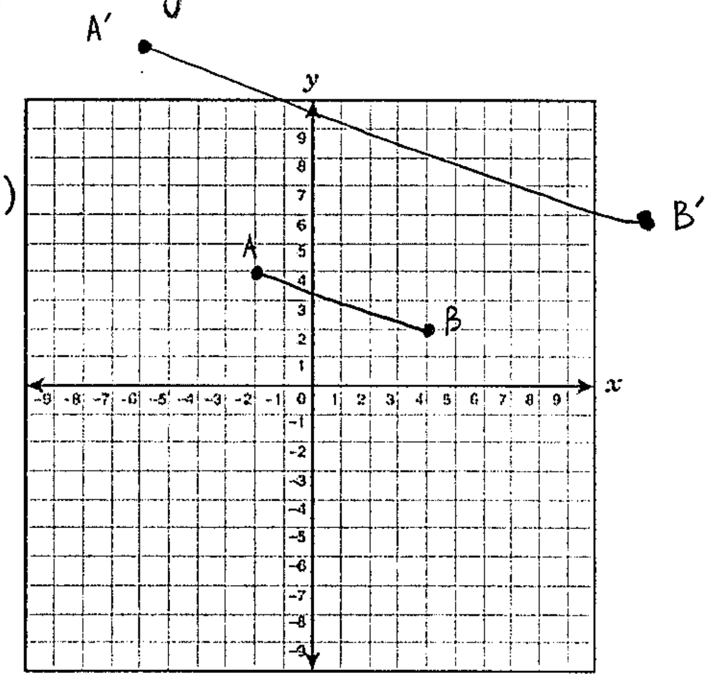
Lesson 12-1: The Transformation of S_k

Vocabulary

Properties of a Dilation (S_k) Theorem: S_k maps (x, y) onto (kx, ky) where k = magnitude. A line & its image are // & the distance between the 2 points is k times the distance between their preimages.

Practice

1. Graph the segment AB, where
A = (-2, 4) and B = (4, 2). $B' = (4 \cdot 3, 2 \cdot 3) = (12, 6)$
$A' = (-2 \cdot 3, 4 \cdot 3) = (-6, 12)$
Graph A'B' under a dilation where the image of (x, y) is (3x, 3y) or S_3.



What is the relationship between $\overline{AB}$ and $\overline{A'B'}$?

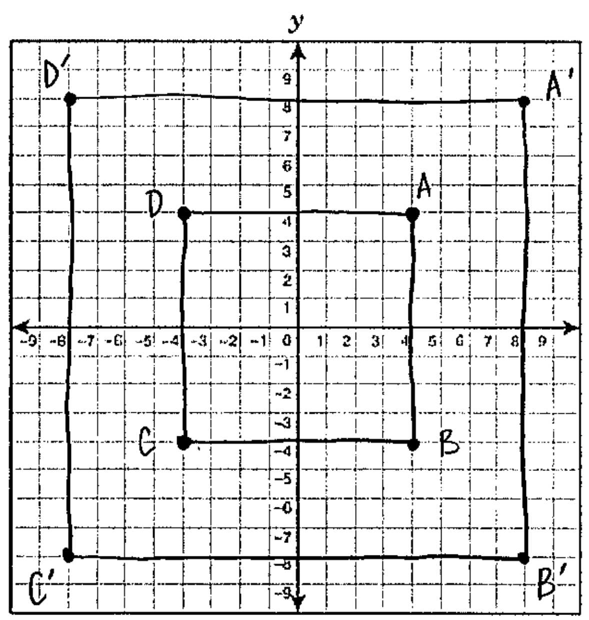
$\overline{AB} \parallel \overline{A'B'}$

How are AB and A'B' related?

A'B' is 3x the length of AB

2. Graph the square ABCD, A = (4, 4), B = (4, -4), C = (-4, -4), and D = (-4, 4).

Graph A'B'C'D' under S_2.



What is the relationship between $\overline{AB}$ and $\overline{A'B'}$?

$\overline{AB} \parallel \overline{A'B'}$

How are AB and A'B' related?

A'B' is 2x length of AB

Lesson 12-2: Properties of Dilations

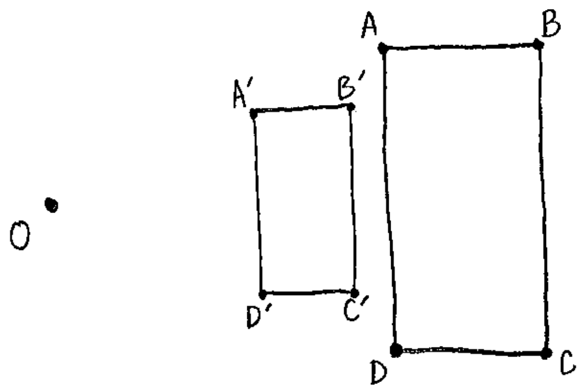
Vocabulary

Dilation Preservation Properties Theorem: every dilation preserves (1) angle measure, (2) betweenness, (3) collinearity

Expansion	figure gets larger	$k > 1$
Contraction	figure gets smaller	$k < 1$
Identity	figure stays the same	$k = 1$

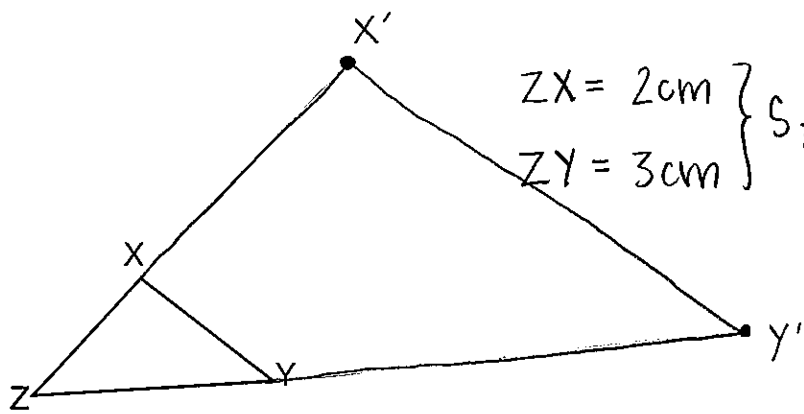
Practice

1. Give the dilation of ABCD under magnitude 0.6 and center O.



$$\left.\begin{array}{l}\overline{OA} = 5cm\\ \overline{OB} = 7cm\\ \overline{OC} = 7cm\\ \overline{OD} = 5cm\end{array}\right\} S_{0.6} \left\{\begin{array}{l}\overline{OA'} = 3cm\\ \overline{OB'} = 4.2cm\\ \overline{OC'} = 4.2cm\\ \overline{OD'} = 3cm\end{array}\right.$$

2. Give the dilation of ABCD under magnitude 3 and center Z.



$$\left.\begin{array}{l}ZX = 2cm\\ ZY = 3cm\end{array}\right\} S_3 \left\{\begin{array}{l}\overline{ZX'} = 6cm\\ \overline{ZY'} = 9cm\end{array}\right.$$

Lesson 12-3: More Properties of Dilations

Vocabulary

Dilation Distance Theorem: under a dilation w/ magnitude $k > 0$, the distance between any 2 image points is k times the distance between their images.

k: the ratio of the length of an image to its preimage

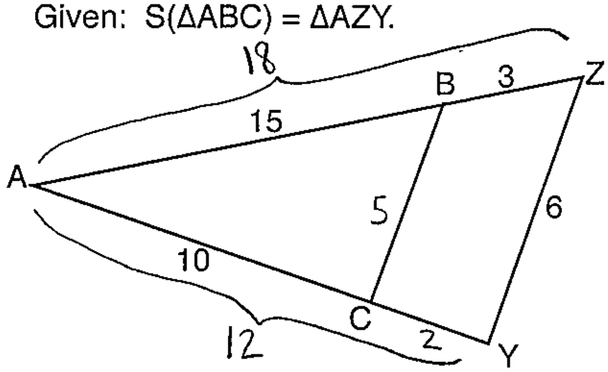
$$k = \frac{A'B'}{AB} = \frac{\text{image}}{\text{preimage}}$$

Practice

1. Name three properties preserved by dilations.

angle measure, betweeness, collinearity

2. a) Find k, the magnitude of the size change.
Given: S(ΔABC) = ΔAZY.



$\frac{AZ}{AB} = \frac{18}{15} = 1.2$, so $k = 1.2$

b) Find as many other lengths as you can. $AC = 10 \cdot 1.2$, so $AY = 12$
$ZY = 6 \div 1.2 = 5$, so $BC = 5$

3. If a segment AB is 12 in and segment A'B' is 6 in, what is the magnitude (k) of the dilation?

$$k = \frac{6}{12} = \boxed{0.5}$$

4. Segment ZY is 20 cm and segment Z'Y' is 4 cm, what is the magnitude (k) of the dilation?

$$k = \frac{4}{20} = \boxed{0.2}$$

Lesson 12-4: Proportions

Vocabulary

Ratio: a quotient of two numbers, $\frac{m}{n}$ or m/n or $m:n$, where m & n are the same quantity

Example: $\frac{6\text{ in}}{7\text{ in}}$ or $\frac{500\text{ ft}}{100\text{ ft}}$ or $\frac{10{,}000\text{ people}}{8\text{ people}}$

Rate: a quotient of two #'s, $\frac{m}{n}$ or m/n or $m:n$, where m & n are different quantities.

Example: $\frac{6\text{ in}}{2\text{ ft}}$ or $\frac{500\text{ ft}}{100\text{ in}}$ or $\frac{100\text{ miles}}{2\text{ hours}}$

Proportion: a statement that two quotients are equal (ex: $\frac{1}{3} = \frac{3}{9}$ or $\frac{4}{10} = \frac{2}{5} = \frac{8}{20}$).

1st Term $\rightarrow a$, 2nd Term $\rightarrow b$, 3rd Term $\rightarrow c$, 4th Term $\rightarrow d$

$$\frac{a}{b} = \frac{c}{d}$$

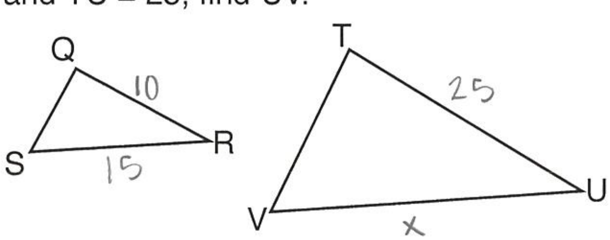
Means-Extremes Property: If $\frac{a}{b} = \frac{c}{d}$, then $ad = bc$.

$\frac{a}{b} = \frac{c}{d}$ means

extremes

Practice

1. ΔQRS is the image of ΔTUV under a size change with center O. If QR = 10, RS = 15, and TU = 25, find UV.



$\frac{QR}{TU} = \frac{SR}{UV}$

$\frac{10}{25} = \frac{15}{x}$

$\frac{10x}{10} = \frac{375}{10}$

$x = 37.5$

So UV = 37.5.

Lesson 12-5: Similar Figures

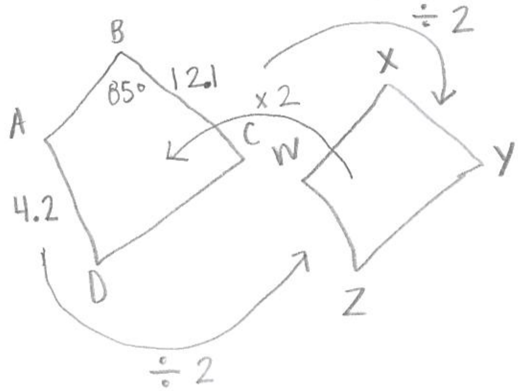
Vocabulary

Similar (~): two figures are similar if they have all = angles & proportional sides.

→ same shape, just different size

Practice

1. ABCD ~ WXYZ. ABCD is the image of WXYZ under S_2. If m∠B = 85, BC = 12.1, and AD = 4.2, find any other angles and side lengths that you can.

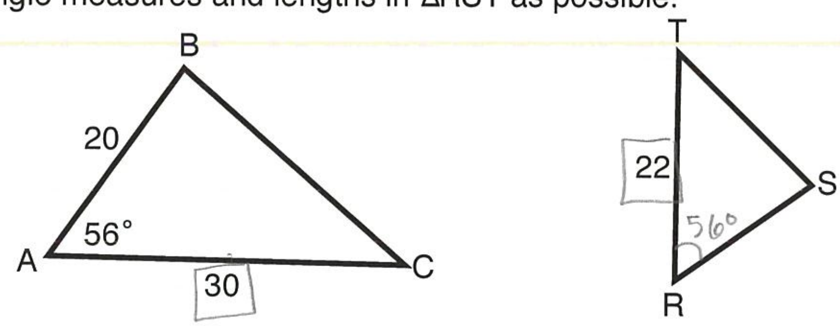


∠X = 85°, XY = 6.05, & WZ = 2.1

4.2 ÷ 2 = 2.1

12.1 ÷ 2 = 6.05

2. ΔABC ~ ΔRST with angle measures and side lengths as indicated. Find as many other angle measures and lengths in ΔRST as possible.



$$\frac{30}{22} = \frac{20}{x}$$

$x \approx 14.7$

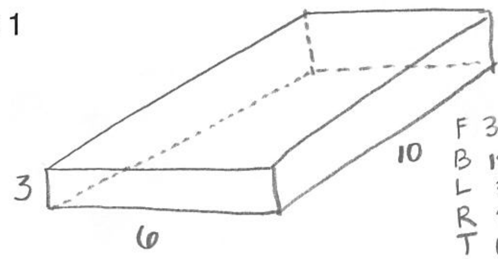
$$\boxed{\begin{array}{l} \angle R = 56° \\ RS = 14.7 \end{array}}$$

Lesson 12-6 & 12-7: Fundamental Theorem of Similarity

Example

Box 1

ea. dimension ×5



F $3 \cdot 6 = 18$
B 18
L $3 \cdot 10 = 30$
R 30
T $6 \cdot 10 = 60$
B 60

216

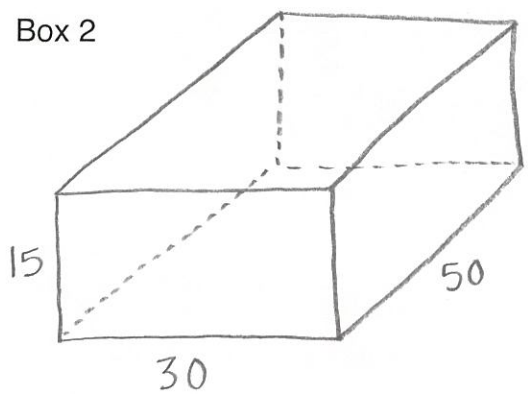
P = $10+6+10+6 = 32$

SA = 216

V = $3 \cdot 6 \cdot 10 = 180$

×5 ×25 ×125

Box 2



F $15 \cdot 30 = 450$
B 450
L $15 \cdot 50 = 750$
R 750
T $30 \cdot 50 = 1500$
B 1500

P = $30+50+30+50 = 160$

SA = 5400

V = $15 \cdot 30 \cdot 50 = 22{,}500$

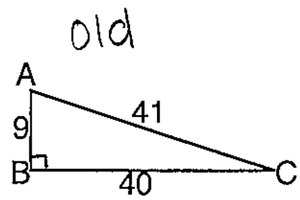
Vocabulary

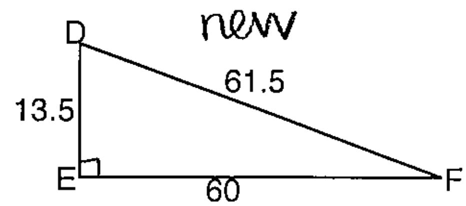
Fundamental Theorem of Similarity: If a figure $G \sim G'$, then lengths are similar by k, areas by k^2, & volumes by k^3

When comparing two figures and their...		
Lengths	**Areas**	**Volumes**
k old side $\cdot k$ = new side	k^2 old area $\cdot k^2$ = new area	k^3 old volume $\cdot k^3$ = new vol.

Practice

1. $\overset{\text{old}}{\Delta ABC} \sim \overset{\text{new}}{\Delta DEF}$





$9+40+41=90$ $\qquad$ $13.5+61.5+60=135$

$\frac{1}{2}\cdot 9\cdot 40=180$ $\qquad$ $\frac{1}{2}\cdot 13.5\cdot 60=405$

a) Give the ratios of the perimeters.

$$\text{new}/\text{old} = 135/90 = \boxed{1.5}\ (k)$$

b) Give the ratios of the areas.

$$\text{new}/\text{old} = 405/180 = \boxed{2.25}\ (k^2)$$

2. A pizzeria sells 12" diameter pizzas with cheese and one topping for \$9.99 (plus tax). The price is based on the amount of ingredients used. What would the charge be for a 14" diameter pizza? area k^2

$$\frac{\text{new length}}{\text{old length}} = \frac{14}{12} = \overset{k}{1.1\overline{6}} \longrightarrow \overset{k^2}{(1.1\overline{6})^2} = \overset{k^2}{1.36} \times \overset{\text{old cost}}{\$9.99} = \overset{\text{new cost}}{\boxed{\$13.60}}$$

3. A deep dish 12" pizza with cheese sells for \$11. What would the charge be for a 15" diameter pizza? volume k^3

$$\frac{\text{new length}}{\text{old length}} = \frac{15}{12} = \overset{k}{1.25} \longrightarrow \overset{k^3}{(1.25)^3} = \overset{k^3}{1.95} \times \overset{\text{old cost}}{\$11} = \overset{\text{new cost}}{\boxed{\$21.45}}$$

4. An octagon has area 150 and longest side length 10. A similar octagon has longest side 4. What is the area of the similar octagon? k^2

$$\frac{\text{new length}}{\text{old length}} = \frac{4}{10} = \overset{k}{.4} \longrightarrow \overset{k^2}{(.4)^2} = \overset{k^2}{.16} \times \overset{\text{old area}}{150} = \overset{\text{new area}}{\boxed{8}}$$

5. A pool with diameter 25 feet holds 12,000 gallons of water. A similar pool holds 2,592 gallons of water. What is the diameter of the similar pool?

$$\frac{\text{new vol.}}{\text{old vol.}} = \frac{2{,}592}{12{,}000} = \overset{k^3}{.216} \longrightarrow \overset{k}{\sqrt[3]{.216}} = \overset{k}{.6} \times \overset{\text{old diam.}}{25} = \overset{\text{new diam.}}{\boxed{15}}$$