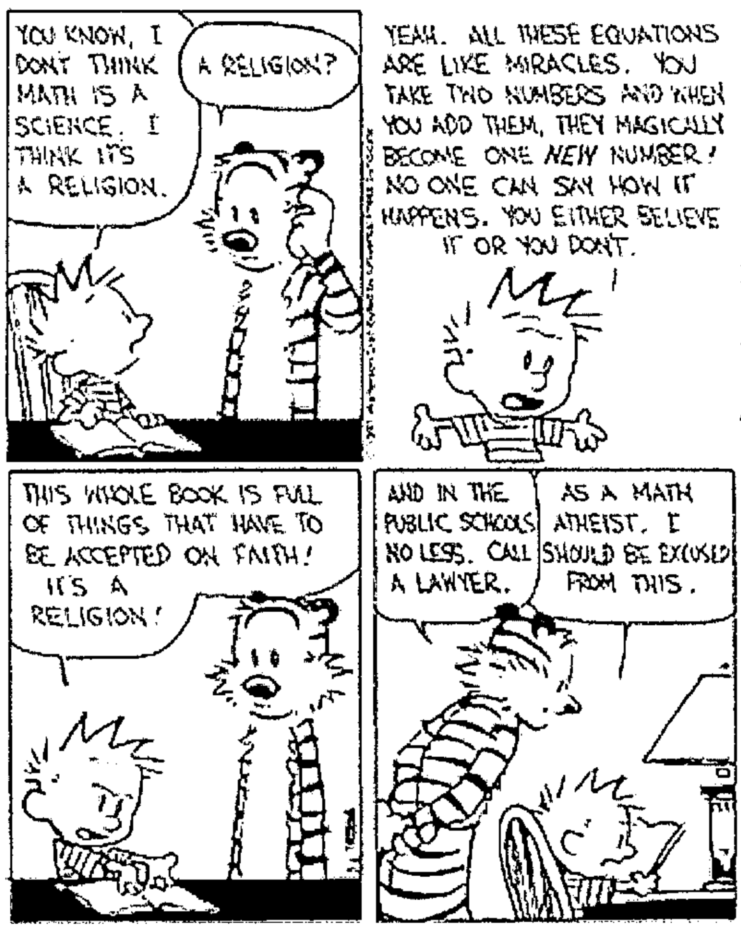
Name: KEY!

Hour: ______

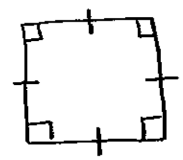
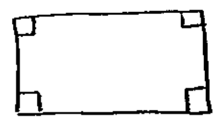
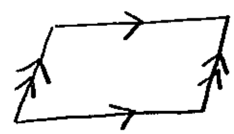
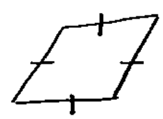
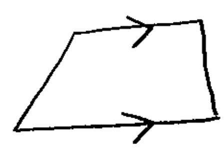
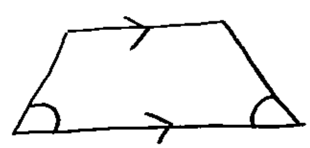
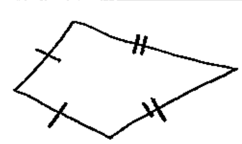
Unit E: Polygons

Geometry 1st Semester



Lesson 6-3: Types of Quadrilaterals

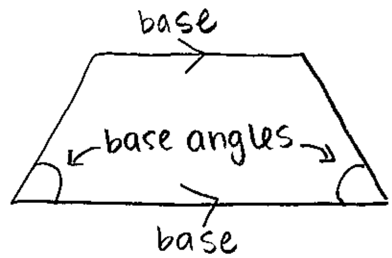
Vocabulary

Name & Description	Picture
Square 4 right angles & 4 equal sides	
Rectangle 4 right angles	
Parallelogram both pairs of opposite sides are parallel	
Rhombus 4 equal sides	
Trapezoid at least one pair of parallel sides	
Isosceles Trapezoid one pair of parallel sides & base angles are congruent	
Kite two consecutive pairs of congruent sides	

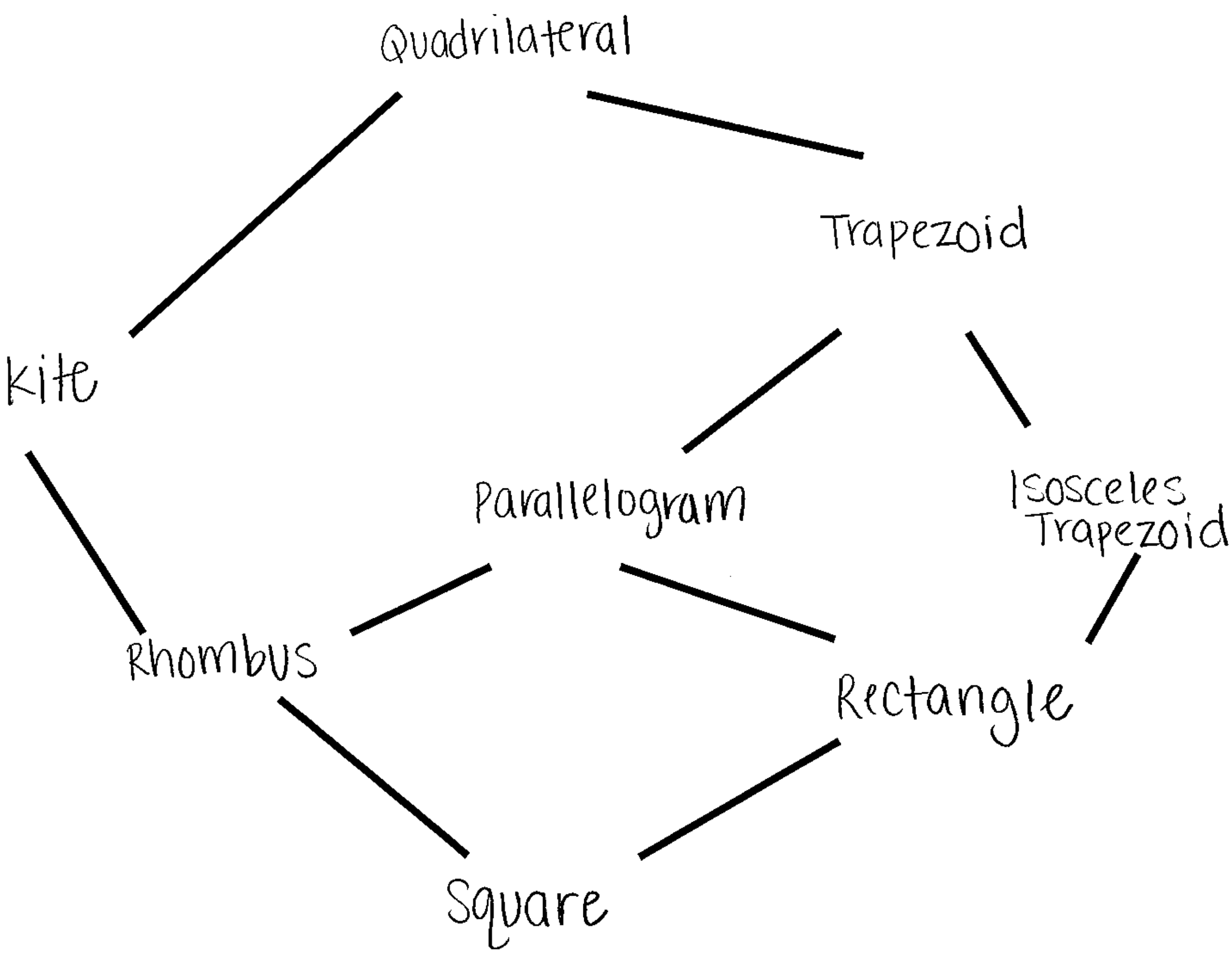
Bases: parallel sides of a trapezoid

Base Angles: two consecutive angles whose vertices are endpoints of a base

Example



QUADRILATERAL HIERARCHY



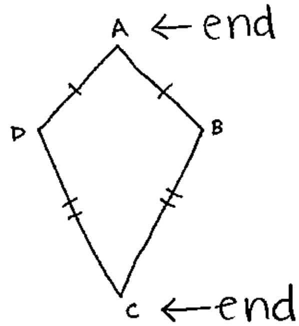
Lesson 6-4: Properties of Kites

(Notice: all 7 types of quadrilaterals are either kites, trapezoids, or both!)

Vocabulary

Ends: the common endpoints of the equal sides

Example

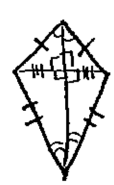


Kite Symmetry Theorem: the line containing the ends of a kite is a symmetry line for the kite

Symmetry Diagonal: the diagonal determined by the ends is the symmetry diagonal

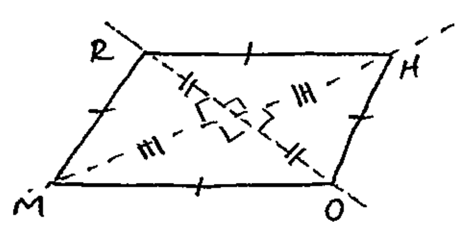
Kite Diagonal Theorem: the symm. diagonal of a kite is the $\perp$ bisector of the other diagonal & bisects the 2 angles @ the ends of the kite.

Example



Rhombus Diagonal Theorem: each diagonal of a rhombus is the $\perp$ bisector of the other diagonal.

Example



Practice

1. What other quadrilaterals are considered kites? These figures will also have the same qualities about them that a kite does. (Hint: think about the hierarchy!)

   rhombus & square

2. Given KITE below with ends *K* and *T*, *EL* = 10, m∠EKT = 43 and m∠ITK = 24.

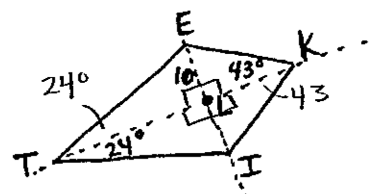
   a. m∠IKT = 43°

   b. m∠ETK = 24°

   c. LI = 10

   d. m∠TIK = 113° ← 180-24-43 = 113

   e. m∠KLE = 90°



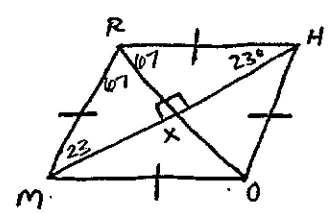
3. Given RHOM at the right.

   a. m∠MHO = 23°

   b. m∠RMH = 23°

   c. m∠OMH = 23°

   d. m∠XRH = 67°



Lesson 6-5: Properties of Trapezoids

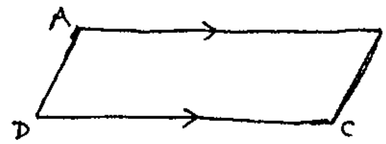
(Notice: all 7 types of quadrilaterals are either kites, trapezoids, or both!)

Vocabulary

Trapezoid Angle Theorem: consecutive angles between a pair of // sides are supplementary

↑ add to 180°

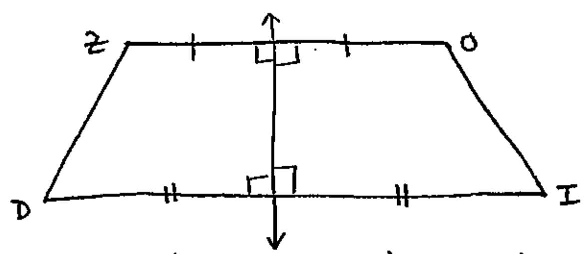
Example



$\angle A + \angle D = 180°$

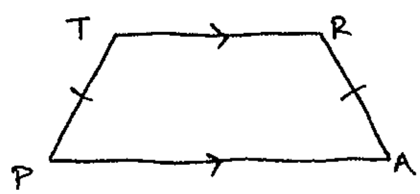
Isosceles Trapezoid Symmetry Theorem: the ⊥ bisector of a base is a ⊥ bisector for the other base & a symmetry line for the figure

Example



Isosceles Trapezoid Theorem: in an isosceles trapezoid, the non-base sides are ≅.

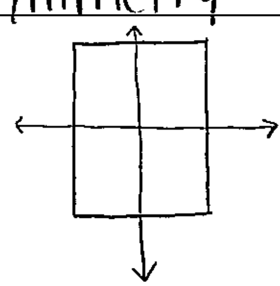
Example



$TP \cong RA$

Rectangle Symmetry Theorem: the ⊥ bisectors of the sides of a rectangle are symmetry lines for the rectangle.

Example

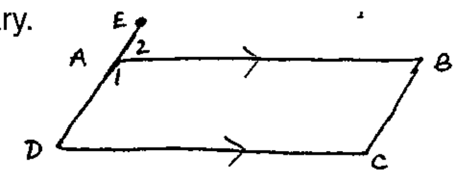


Practice

1. What other quadrilaterals are considered trapezoids? These figures will also have the same qualities about them that a trapezoid does. (Hint: think about the hierarchy!)

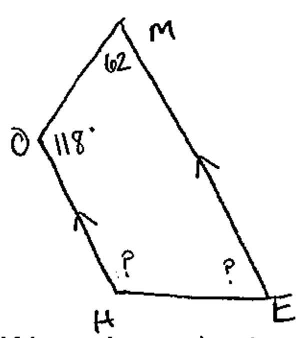
parallelogram, isosceles trapezoid, rectangle, square

2. Given: *ABCD* is a trapezoid with *AB* // *CD* and *AD* has been extended to point E.
Prove: ∠1 and ∠D are supplementary.



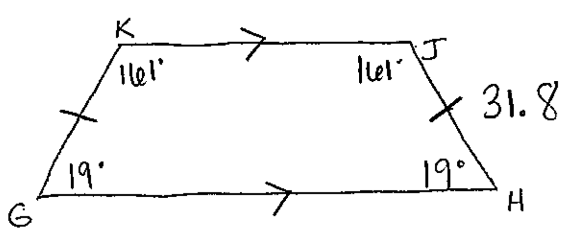
Conclusions	Justifications
0. ABCD is trap. w/ AB//CD & AD is extended to point E	Given
1. $m\angle 1 + m\angle 2 = 180$	Supplementary angles
2. $m\angle 2 = m\angle D$	Corresponding Angles
3. $m\angle 1 + m\angle D = 180$	Substitution
4. ∠1 & ∠D are supplementary	Definition of supplementary angles

3. In trapezoid HOME, HO // ME. If $m\angle O = 118$, find the measures of as many other angles as you can.



$\angle M = 62°$

4. GHJK is an <u>isosceles</u> trapezoid with bases GH and JK, where HJ = 31.8 and $m\angle H = 19$. Find as many other lengths and angle measures as possible.



GK = 31.8

$\angle G = 19°$

$\angle J = 161°$

$\angle K = 161$

Lesson 7-7 & 7-8: Properties of Parallelograms

(Notice: there are 3 other quadrilaterals under a parallelogram in the hierarchy!)

Vocabulary

Properties of a Parallelogram Theorem: In any parallelogram,

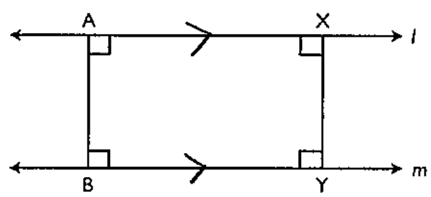
a) opposite sides are congruent

b) opposite angles are congruent

c) the diagonals intersect @ their midpoints

Parallel Lines Distance Theorem: the distance between two given parallel lines is constant.

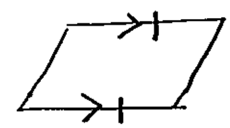
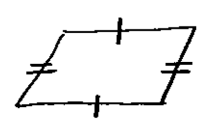
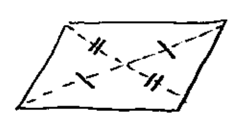
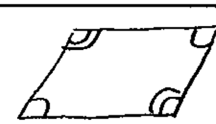
<u>Example</u>



$AB \cong XY$

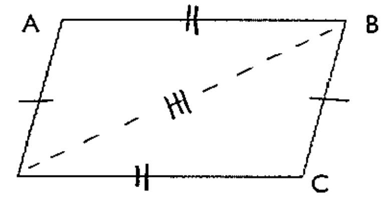
Parallelogram Symmetry Theorem: every parallelogram has 2-fold rotation symmetry about the intersection of its diagonals.

Sufficient Conditions for a Parallelogram Theorem: If, in a quadrilateral,

	a)	one pair of sides is both // & $\cong$	
<u>or</u>	b)	both pairs of opposite sides are $\cong$	
<u>or</u>	c)	the diagonals bisect each other	
<u>or</u>	d)	both pairs of opposite angles are $\cong$	

then the quadrilateral is a parallelogram.

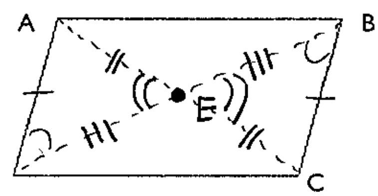
Practice



1. Given: ABCD is a parallelogram.
Prove: In any parallelogram, opposite angles are congruent.

Conclusions	Justifications
0. ABCD is a parallelogram	Given
1. AD ≅ BC, AB ≅ DC	Sufficient Condition for Parall.
2. BD ≅ BD	Reflexive Property
3. ΔABD ≅ ΔCDB	SSS
4. ∠DAB ≅ ∠BCD	CPCF

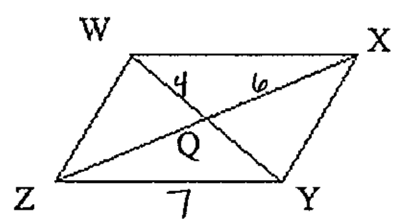
same as "To Prove"



2. Given: ABCD is a parallelogram.
Prove: In any parallelogram, the diagonals intersect at their midpoints.

Conclusions	Justifications
0. ABCD is a parallelogram	Given
1. ∠ADB ≅ ∠CBD	AIA
2. ∠AED ≅ ∠CEB	Vertical Angles
3. AD ≅ BC	Sufficient Condition for Parallel.
4. ΔADE ≅ ΔCBE	AAS
5. AE ≅ CE, DE ≅ BE	CPCF
6. E is the midpoint of AC and BD	Definition of midpoint

3. In parallelogram WXYZ, WQ = 4, XQ = 6, and YZ = 7.



a. Find QY. 4

b. Find WX. 7

c. Name both pairs of parallel sides.

WX // YZ & WZ // XY

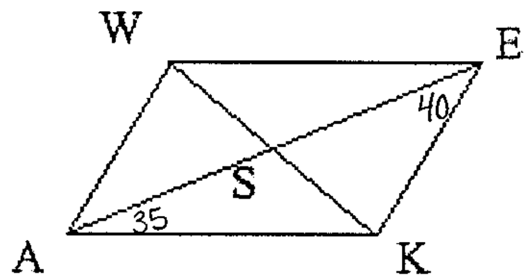
4. Refer to the quadrilateral below. If m∠KEA = 40 and m∠EAK = 35, find the following angles.

a. ∠EAW 40°

b. ∠AEW 35°

c. ∠EWA 105°

d. ∠EKA 105°

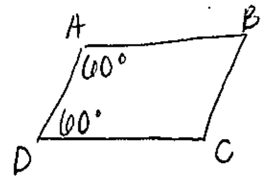


180 - 75

5. Given the following information, is ABCD a parallelogram?

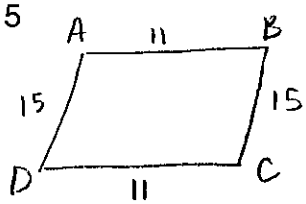
a. m∠BAD = 60, m∠ADC = 60

No, n.e.i.



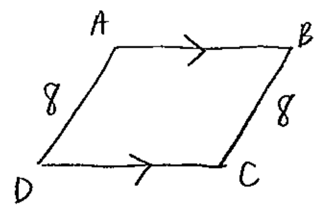
b. AB = 11, BC = 15, CD = 11, AD = 15

Yes, opposite sides are ≅.



c. AB // CD, AD = 8, BC = 8

No, n.e.i.



Lesson 5-7: Sums of Angle Measures in Polygons

Vocabulary

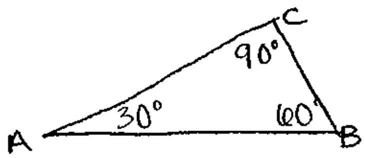
Triangle Sum Theorem: the sum of all of the angles of a triangle is 180°.

Quadrilateral Sum Theorem: the sum of all of the angles of a convex quadrilateral is 360°.

Polygon Sum Theorem: the sum of all of the angles of a convex n-gon is $(n-2)\cdot 180$.

Practice

1. In ABC, the angles are in the ratio 1:2:3. Find m∠A, m∠B, and m∠C.



$1x + 2x + 3x = 180$

$\frac{6x}{6} = \frac{180}{6}$

$x = 30$

$1x = 30$

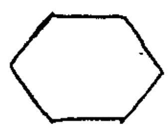
$2x = 60$ & $3x = 90$

2. How many degrees are in a 5 and 6 sided figure?

$(n-2)\cdot 180$

$(5-2)\cdot 180$

$3\cdot 180 = \boxed{540°}$



$(n-2)\cdot 180$

$(6-2)\cdot 180$

$4\cdot 180 = \boxed{720°}$

3. If the measure of the angles of a quadrilateral are in the ratio 2:3:4:6, what are the measures of the angles?

360°

put x w/ it & add

$2x + 3x + 4x + 6x = 360$

$\frac{15x}{15} = \frac{360}{15}$

$x = 24$

$2x = 2\cdot 24 = \boxed{48}$

$3x = 3\cdot 24 = \boxed{72}$

$4x = 4\cdot 24 = \boxed{96}$

$6x = 6\cdot 24 = \boxed{144}$

4. Give the sum of the measures of the angles of a convex octagon.

8 sides

$(n-2)\cdot 180$

$(8-2)\cdot 180$

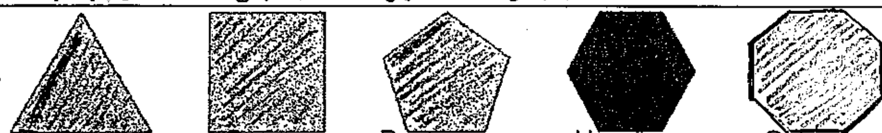
$6\cdot 180 = \boxed{1080°}$

Lesson 6-7: Regular Polygons

Vocabulary

Regular Polygon: a convex polygon whose angles are all ≅ & whose sides are all ≅

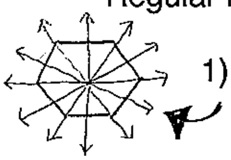
Example



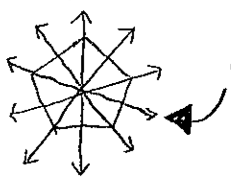
Equi<u>lateral</u>: all the sides of a polygon are ≅

Equi<u>angular</u>: all the angles of a polygon are ≅

Regular Polygon Symmetry Theorem: every regular *n*-gon possesses,



1) (if *n* is even): n symm. lines which are ⊥ bisectors of the sides, & then the bisectors of the angles

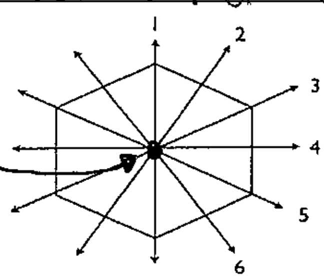


(if *n* is odd): n symm. lines which each bisect an angle & are a ⊥ bisector for the opposite side.

2) n-fold rotational symmetry

Example

center



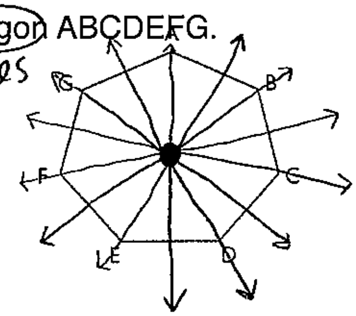
Regular Hexagon
* 6 symm. lines
* 6-fold rotational symm.

Practice

1. a. Find the number of *n*-fold rotations of a regular heptagon ABCDEFG.
   7-fold
   7 sides

   b. Draw all lines of symmetry on heptagon ABCDEFG.

   7 symm. lines



2. Find the measure of an interior angle of a:

a) regular octagon

$(8-2)\cdot 180$

$6\cdot 180 = \frac{1{,}080}{8} = \boxed{135^\circ}$

b) regular decagon (10)

$(10-2)\cdot 180$

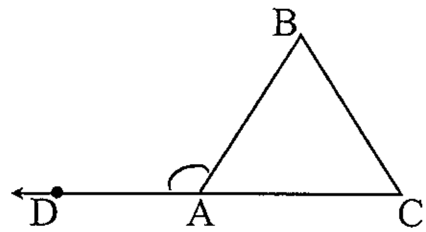
$8\cdot 180 = \frac{1{,}440}{10} = \boxed{144^\circ}$

Lesson 7-9: Exterior Angles

Vocabulary

Exterior Angle: the angle created when one side of a figure is extended

Example



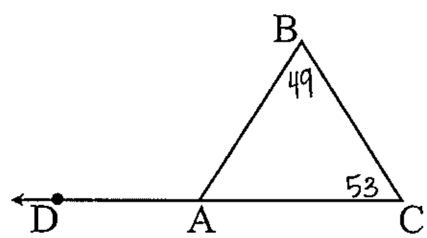
$\angle BAD$ is exterior

Exterior Angle Theorem: In a $\triangle$, the measure of an exterior angle is = to the sum of the other 2 interior angles

so $\angle BAD = \angle B + \angle C$

Practice

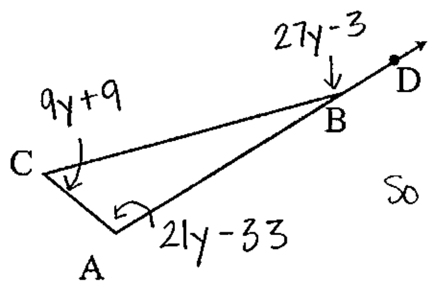
1. Give the measures of $\angle BAD$ if $\angle B = 49$ and $\angle C = 53$.



$49 + 53 = 102°$

So, $\angle BAD = 102°$

2. Find y if $\angle BAC = 21y - 33$, $\angle ACB = 9y + 9$ and $\angle CBD = 27y - 3$.



$\underbrace{21y - 33} + \underbrace{9y + 9} = 27y - 3$

$30y - 24 = 27y - 3$
$-27y \qquad -27y$

$3y - 24 = 3$
$+24 \qquad +24$

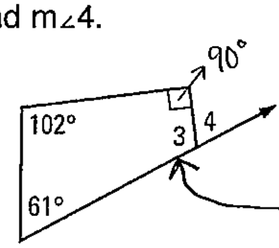
$\frac{3y}{3} = \frac{27}{3}$

$\boxed{y = 9}$

So $y = 9$.

3. Find $m\angle 3$ and $m\angle 4$.

Not a $\triangle$, so can't add up the others!



$360 - 102 - 61 - 90 = 107°$, $\boxed{\text{so } \angle 3 = 107}$

Then $180 - 107 = 73°$, $\boxed{\text{so } \angle 4 = 73}$