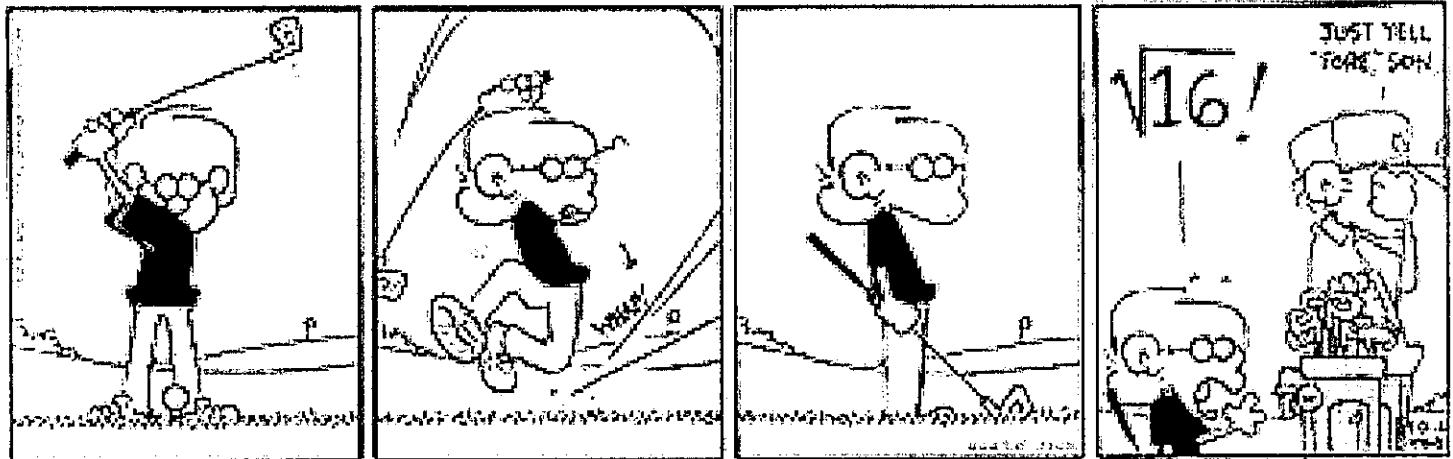


Name: KEY!

Hour: _____

Unit D: Transformations

Geometry 1st Semester



Lesson 4-1: Reflecting Points**Vocabulary**Preimage: the original image/pictureTransformation: a correspondence between two sets of points, such that:

- "mapping"
1. Each point of the preimage has a unique image.
 2. Each point of the image has a unique preimage.

Notation

$T(P)$	the image of point P, under the transformation T ; "T of P"
$r(A) = A'$	the reflection image of A is A'
$r_m(P) = Q$	the reflection image of P over line m is Q

Examples

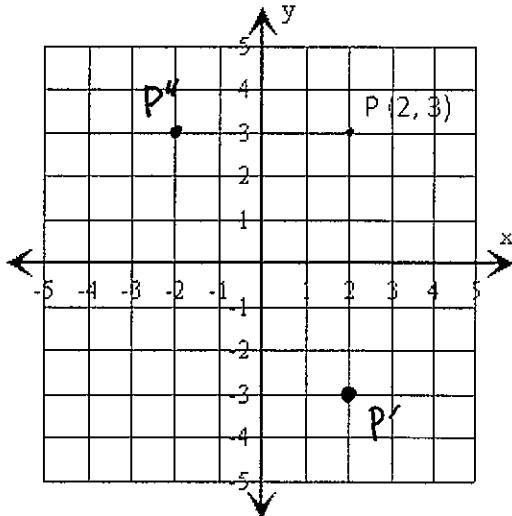
Reflecting points over the x and y-axis:

1. Using your MIRA, reflect point P over the x-axis. Label the new point P' . Then, reflect point P' over the y-axis. Label the new point P'' .

2. What are the coordinates of the new points?

$$P' = (2, -3)$$

$$P'' = (-2, 3)$$

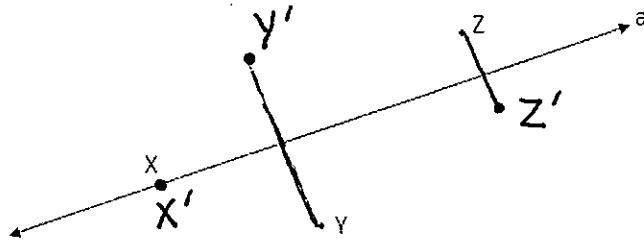


What do you notice about the x and y values when you reflect them over a particular axis?

Reflect over x-axis: y coordinate is opposite

Reflect over y-axis: x coord. is opposite

- 3.
- Draw $r_a(X)$. Label it X' .
 - Draw $r_a(Y)$. Label it Y' .
 - Draw $r_a(Z)$. Label it Z' .



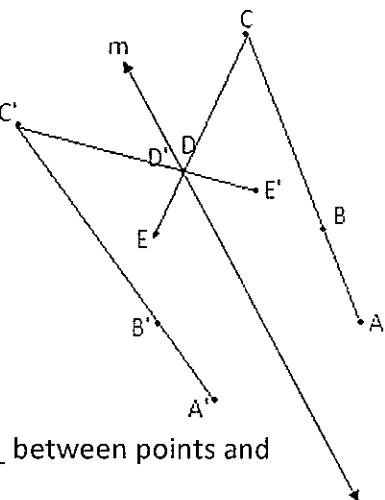
4. Draw a line connecting Y to Y' and then a line connecting Z to Z' ...what is the relationship between each line that you drew and line a ?

"a" is a perpendicular bisector to the segment $\overline{YY'}$ & $\overline{ZZ'}$.

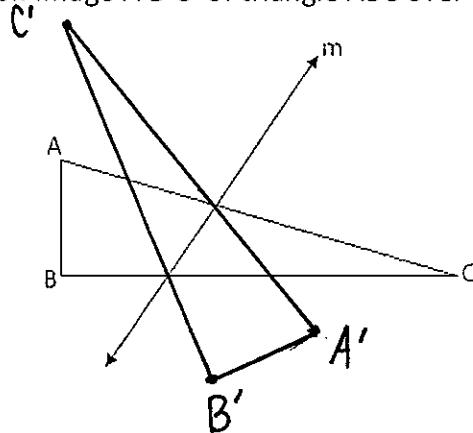
Lesson 4-2: Reflecting Figures**Vocabulary****Reflection Postulate**

Under a reflection:

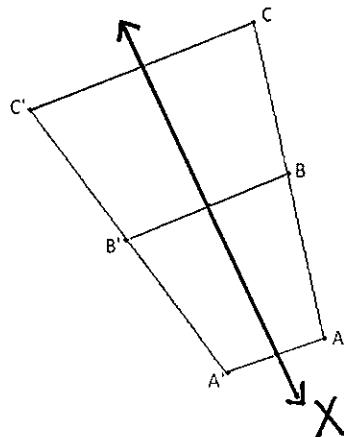
- A. Angle measure is preserved.
- B. Betweenness is preserved.
- C. Collinearity is preserved.
- D. Distance is preserved.
- e. Orientation is REVERSED.
- f. There is a 1-1 correspondence between points and their images.

**Examples**

1. Draw the reflection image $A'B'C'$ of triangle ABC over line m .



2. Draw line x so that $r_x(ABC) = A'B'C'$.

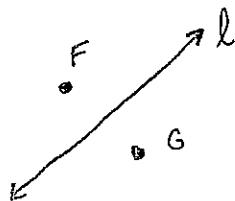


Lesson 6-1: Reflection-Symmetric Figures

Vocabulary

Reflection-Symmetric Figure: if & only if there is a line m such that $r_m(F) = F$, the line m is a symmetry line.

Flip-Flop Theorem: If F & G are points/figures & $r_l(F) = G$, then $r_l(G) = F$.



Segment Symmetry Theorem: every segment has 2 symm. lines:

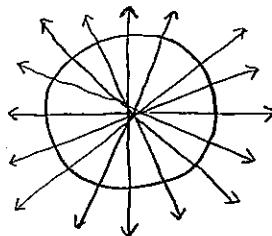
- it's perpendicular bisector &
- the line containing the segment

Angle Symmetry Theorem: the bisector of an angle is the symmetry line of an angle.

Side Switching Theorem: If one side of an angle is reflected over the symm. line, its image is the other side.

Circle Symmetry Theorem: a circle is reflection-symm. to any line through its center.

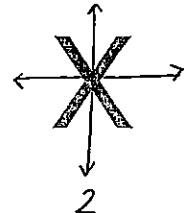
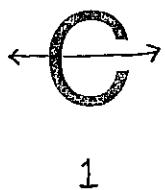
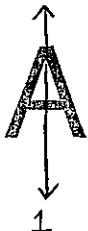
Example



Symmetric Figures Theorem: If a figure is reflection Symm., then any pair of corresponding parts under the symmetry are congruent.

Practice

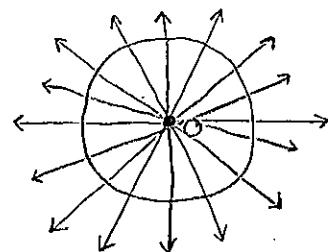
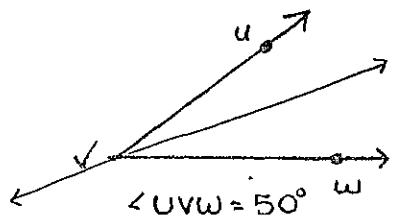
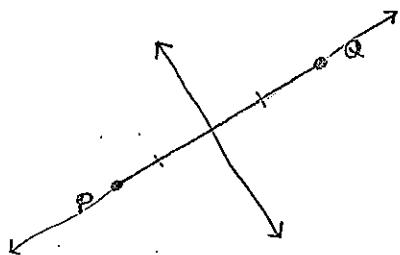
1. Draw all of the lines of symmetry for the letters below.



N

none

2. Draw all of the lines of symmetry for the figures below.



infinitely
many

3. For the figure below, complete the following:

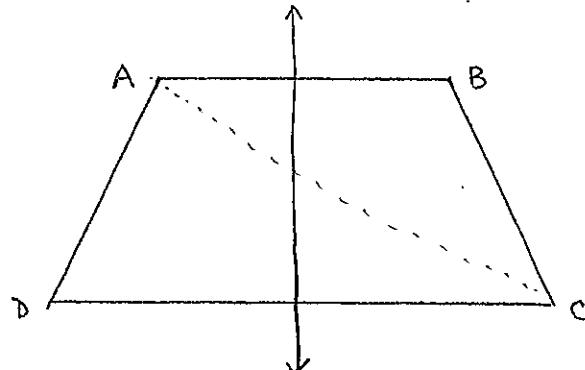
a. Draw the line of symmetry.

b. $\angle A = \angle \underline{B}$

c. $\angle D = \angle \underline{C}$

d. $BC = \underline{AD}$

e. $AC = \underline{BD}$



Lesson 4-4: Translations

Vocabulary

Composite: $T \circ S$, composite of first transformation S & then T ; maps each point P onto $T(S(P))$

Alternative Notations for Compositions:

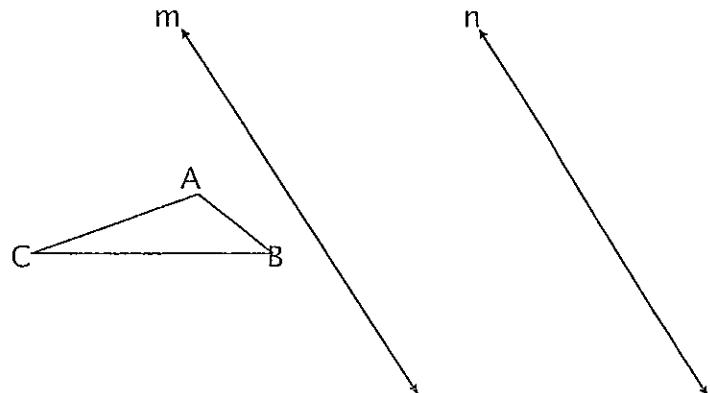
$$r_m(r_n(\Delta ABC)) = A''B''C'' \quad \text{OR} \quad r_m \cdot r_n(\Delta ABC) = A''B''C''$$

Translation/Slide: composite of 2 reflections over parallel lines

Properties of Translations:

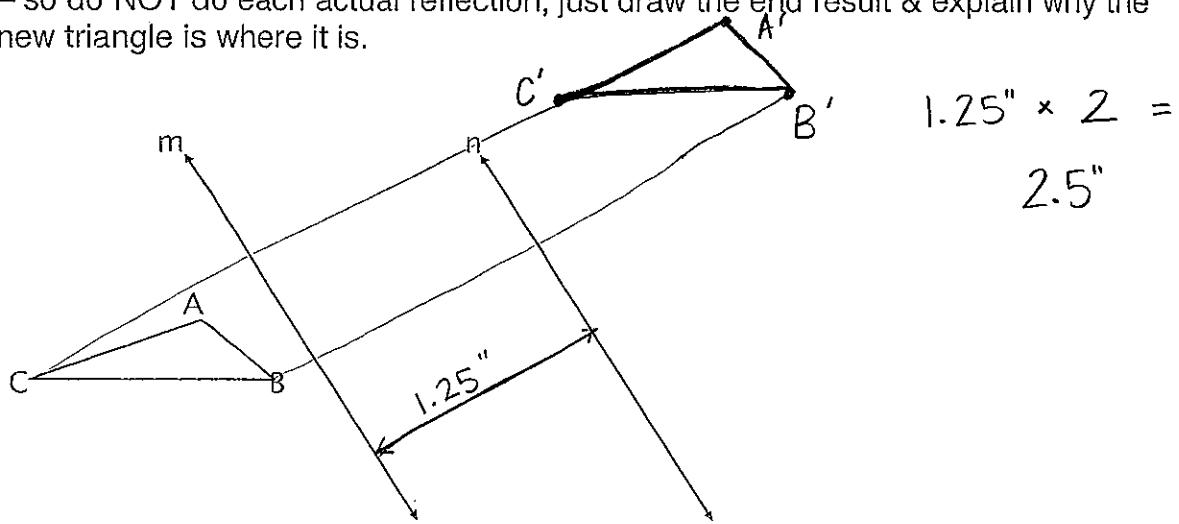
- ABCD & 1. Preserved: angle measure, betweenness, collinearity, distance, & orientation
- 0
2. Direction is given by any ray from a preimage point through its image point.
3. Magnitude: the distance between the original & new is $2 \times$ distance bet. the lines

Two Reflections Theorem: If $m \parallel n$, the translation $r_m \cdot r_n$ has magnitude $2 \times$ the distance bet. n & m , in direction of L to n & m



Practice

1. In the figure below, $m \parallel n$. Use the Two Reflection Theorem to draw $r_n \cdot r_m(\Delta ABC)$ – so do NOT do each actual reflection, just draw the end result & explain why the new triangle is where it is.



The new figure, $A'B'C'$, is $2.5"$ from the old figure, ABC , in the direction of $m \perp n$

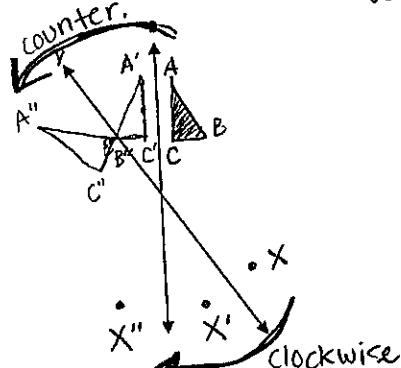
Lesson 4-5: Rotations

Vocabulary

Rotation: the composite of 2 reflections over intersecting lines.

Rotations Preserve: Angle measure, Betweenness, A, B, C, D & O! Collinearity, Distance, & Orientation

Clockwise vs. Counterclockwise: *shortest path between image & pre-image!

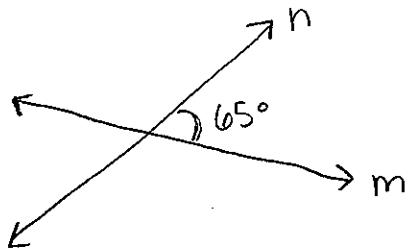


$r_y \circ r_z$ OR $r_y(r_z(\Delta ABC))$: counterclockwise
positive degrees

$r_z \circ r_y$ OR $r_z(r_y(\cdot X))$: clockwise
negative degrees

Two Reflections Theorem for Rotations: If m intersects n , the rotation $r_m \circ r_n$ has center O , where m intersects n and the magnitude of the rotation is 2 × the measure of the smaller angle made by the intersecting lines n & m

Ex:



Magnitude :

$$2 \cdot 65^\circ = 130^\circ$$

+130° if went counterclockwise

-130° if went clockwise

Practice

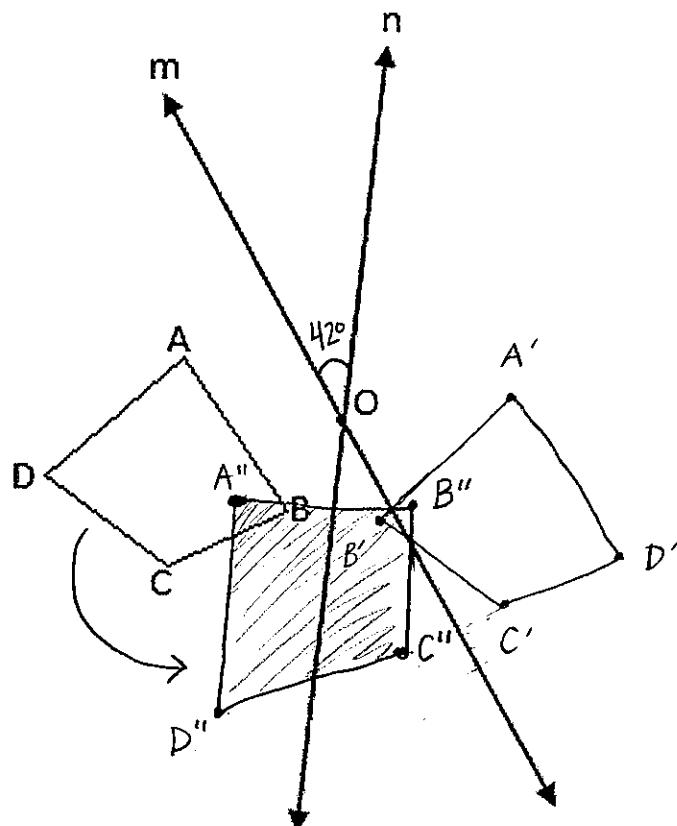
1. For the image below, reflect $r_m(r_n(ABCD))$.
 - a. Did the figure rotate clockwise or counterclockwise?

CLOCKWISE

- b. What is the magnitude of the rotation?

$$42^\circ \cdot 2 = 84^\circ$$

So, $\boxed{-84^\circ}$



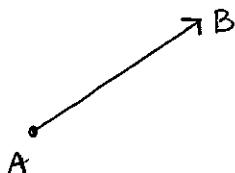
Lesson 4-6: Translations with Vectors

Vocabulary

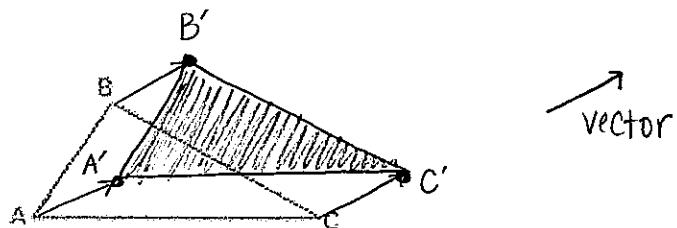
Vector: a quantity that can be characterized by its direction & magnitude (length)

Practice

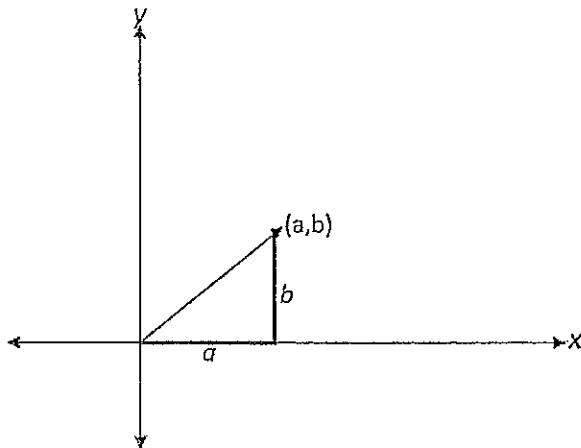
1. Draw the vector \overrightarrow{AB} .



2. Draw the image of $\triangle ABC$ under the translation with the given vector.
(Hint: eyeball it!)



Vectors in the Coordinate Plane



For vector (a, b) shown above, a is the horizontal component and b is the vertical component.

Translations Using Vectors

When translating a figure on the coordinate plane by a vector (a, b) , the image point (x, y) becomes $(x+a, y+b)$.

Practice

just add them together!

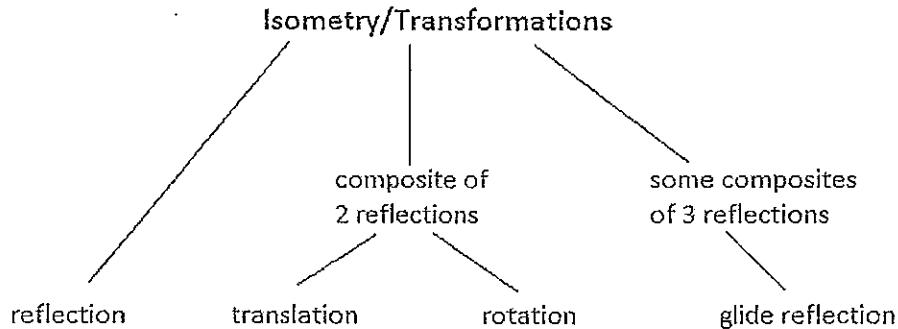
1. A line segment has endpoints $A = (5, -2)$ and $B = (3, 7)$. Find the points of the image under a translation by vector $(-1, 4)$.

$$\begin{aligned}A' &= (5 + -1, -2 + 4) \\&= (4, 2)\end{aligned}$$

$$\begin{aligned}&\quad \& \quad B' = (3 + -1, 7 + 4) \\&\quad \quad \quad = (2, 11)\end{aligned}$$

Transformation in a Plane	Determined By	Description in Terms of Reflections
Reflection	one line	r_m
Rotation	center & magnitude	$r_m \circ r_n$ if m intersects n
Translation	vector	$r_m \circ r_n$ if $m // n$

Lesson 4-7: Isometries & Glide Reflections



Vocabulary

Isometry: a reflection or composite of reflections

↑ aka "transformation"

Concurrent: when 3 or more lines intersect at a point

Glide Reflection: a combination of a reflection over a line and a translation whose direction is parallel to the reflecting line.

Notation

Glide Reflection $G = \begin{matrix} T \\ r_m \end{matrix}$, where r_m is a reflection and T is a translation.
Practice $\begin{matrix} \uparrow & \uparrow \\ \text{second} & \text{first} \end{matrix}$

- Given the figure below, let $G = T \circ r_m(ABCD)$, where m is a line and T is the translation given by vector \vec{XY} . Draw $G(ABCD)$.

