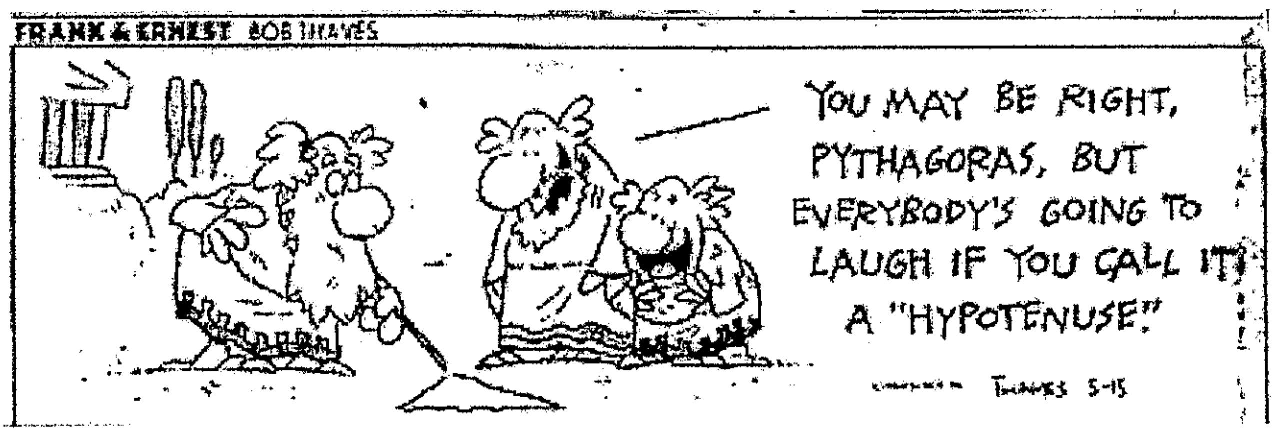
Name: KEY!

Hour: ______

Unit C: Triangles

Geometry 1st Semester

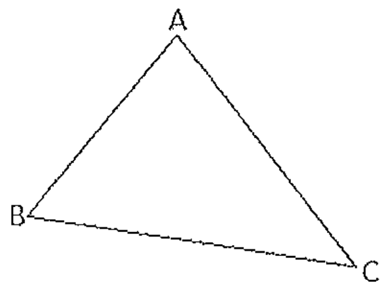


Lesson 2-7: Triangle Inequality

Vocabulary

Triangle Inequality Postulate: the sum of the lengths of any 2 sides is greater than the third side.

So, for triangle ABC below, this postulate means three things:



1. $AB + BC > AC$
2. $AB + AC > BC$
3. $BC + AC > AB$

Practice

1. Is it possible to have a triangle with side lengths 3, 5, 9? Explain...

   $3 + 5 = 8$, no since $8 < 9$.

2. Is it possible to have a triangle with side lengths 3, 7, 9? Explain...

   $3 + 7 = 10$, yes since $10 > 9$.

3. Two sides of a triangle are 15 and 20.

   a. The third side must be longer than what length? Why?

   $20 - 15 = 5$.

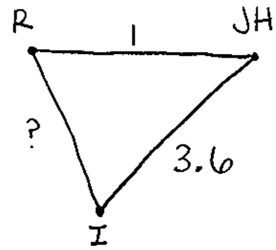
   b. The third side must be shorter than what length? Why?

   $20 + 15 = 35$.

   c. If the third side is called *x*, write an inequality showing its possible lengths.

   $5 < x < 35$

4. The distance from Roberto's house to the Junior High is 1 mile. The distance from Ian's house to the Junior High is 3.6 miles. Write an inequality about the possible distances between Roberto and Ian's houses.



$1 + 3.6 = 4.6$

$3.6 - 1 = 2.6$

$2.6 < x < 4.6$

5. A triangle has two side lengths of 10 and 3.

a. The third side must be longer than what length? Why?

$10 - 3 = 7$

b. The third side must be shorter than what length? Why?

$10 + 3 = 13$

c. If the third side is called *x*, write an inequality showing its possible lengths.

$7 < x < 13$

6. Can a triangle have side lengths:

a. 3cm, 8cm, 6cm? $3 + 6 = 9$, yes since $9 > 8$

b. 3cm, 8cm, 5cm? $3 + 5 = 8$, no since $8 = 8$

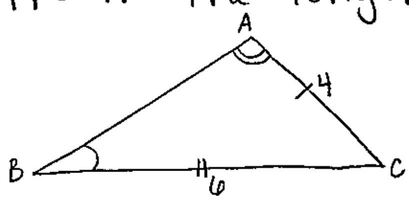
c. 3cm, 3cm, 3cm? $3 + 3 = 6$, yes since $6 > 3$

Lesson 7-9: Unequal Sides & Angles Theorem

Vocabulary

Unequal Sides Theorem: If 2 sides are not ≅, then the angles opposite them are not ≅ & the longer side is opposite the longer angle.

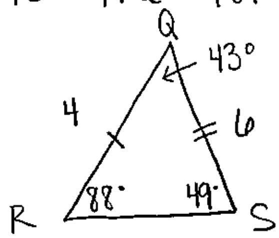
Example



∠A is larger than ∠B

Unequal Angles Theorem: If 2 angles are not ≅, then the sides opposite them are not ≅ & the larger angle is opposite the longer side.

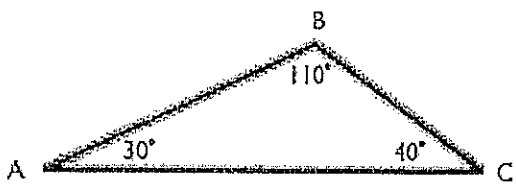
Example



RS < QR < QS

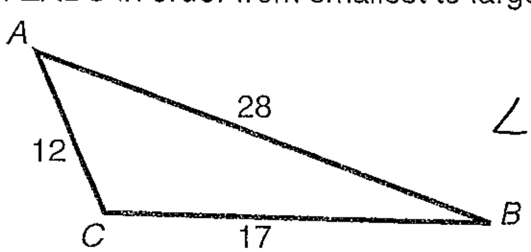
Practice

1. Put the sides of ΔABC in order from smallest to largest.



BC < AB < AC

2. Put the angles of ΔABC in order from smallest to largest.

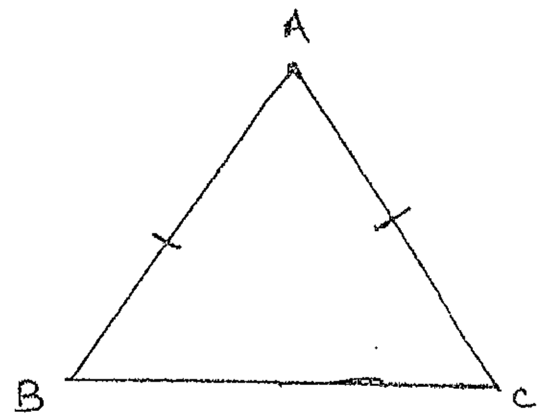


∠B < ∠A < ∠C

Lesson 6-2: Isosceles Triangles

Vocabulary

Parts of an Isosceles Triangle:



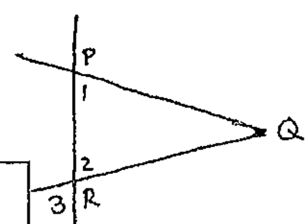
Two Congruent Sides tick marks $\overline{AB}$ & $\overline{AC}$
Vertex Angle between 2 $\cong$ sides, $\angle A$
Base Angles other angles $\angle B$ & $\angle C$
Base non-$\cong$ side $\overline{BC}$

Isosceles Triangle Symmetry Theorem: the line bisecting vertex angle is a symmetry line

Isosceles Triangle Coincidence Theorem: symmetry line is the $\perp$ bisector of base & is the median of base.

* Isosceles Triangle Base Angles Theorem (ITBAT): If a triangle has 2 $\cong$ sides, then the angles opposite them are $\cong$

Practice



1. Given: The figure at the right, with $PQ = QR$.
   Prove: $m\angle 1 = m\angle 3$.

Conclusions	Justifications
1. $PQ = QR$	Given
2. $m\angle 1 = m\angle 2$	ITBAT
3. $m\angle 2 = m\angle 3$	Vertical Angles
4. $m\angle 1 = m\angle 3$	Transitivity

Vocabulary

Equilateral Triangles have... 3 equal sides & 3 lines of symmetry.

Equilateral Triangle Symmetry Theorem: every equil. △ has 3 symm. lines, which are the bisectors of its angles & the ⊥ bisectors of its sides.

Equilateral Triangle Angle Theorem: If a △ is equilateral, then it is equiangular.

Equiangular: all angles have the same measure

Corollary: every equil. △ has angles that each measure 60°.

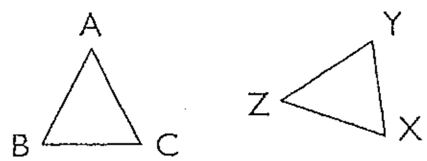
Practice:

Lesson 5-1: Corresponding Parts of Congruent Figures

Vocabulary

Corresponding Parts of Congruent Figures Theorem (CPCF Theorem): If two figures are congruent, then any pair of corresponding parts is congruent.

Example



If $\Delta ABC \cong \Delta ZYX$, then by CPCF:

$\angle A \cong \angle Z$, $\angle B \cong \angle Y$, $\angle C \cong \angle X$

$\overline{AB} \cong \overline{ZY}$, $\overline{BC} \cong \overline{YX}$, & $\overline{AC} \cong \overline{ZX}$

ABCD Theorem: every transformation preserves: **A**ngle measure, **B**etweenness, **C**ollinearity, & **D**istance

Equivalence Properties of Congruence: For any figure F, G, and H:

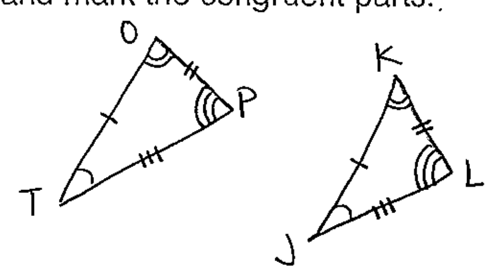
a) $F \cong F$ (Reflexive Property)

b) If $F \cong G$, then $G \cong F$ (Symmetric Property)

c) If $F \cong G$ & $G \cong H$, then $F \cong H$. (Transitive Prop.)

Practice

1. $\Delta TOP \cong \Delta JKL$. List the six pairs of congruent parts. Sketch a possible situation and mark the congruent parts.

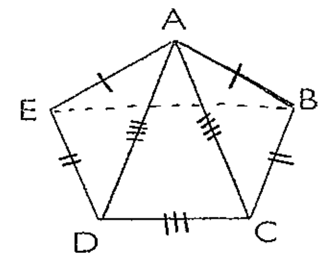


$\angle T \cong \angle J$ $\quad$ $\overline{TO} \cong \overline{JK}$

$\angle O \cong \angle K$ $\quad$ $\overline{OP} \cong \overline{KL}$

$\angle P \cong \angle L$ $\quad$ $\overline{TP} \cong \overline{JL}$

2. ABCD ≅ AEDC.



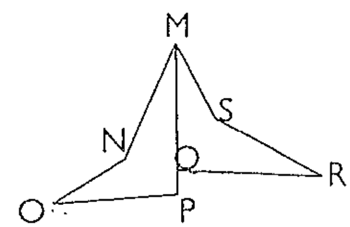
a. List all pairs of congruent parts.

$\angle A \cong \angle A$, $\angle B \cong \angle E$, $\angle C \cong \angle D$,
$\overline{AB} \cong \overline{AE}$, $\overline{BC} \cong \overline{ED}$, $\overline{DC} \cong \overline{CD}$, &
$\overline{AD} \cong \overline{AC}$.

b. Name two triangles which are isosceles.

$\triangle ADC$ & $\triangle AEB$.

3. Suppose MNOP ≅ RSMQ. Using the ABCD Theorem:



a. Which angle has measure equal to m∠PMN.

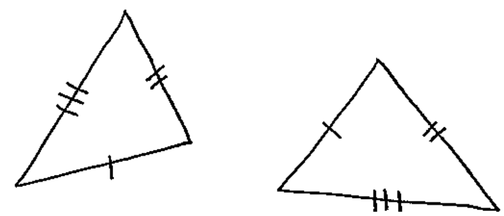
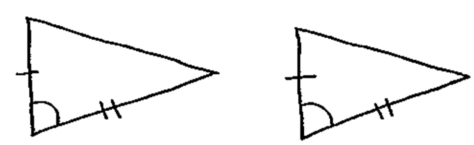
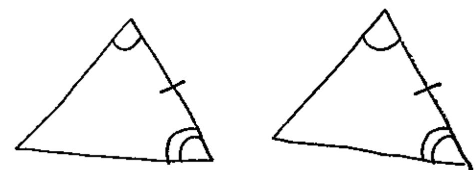
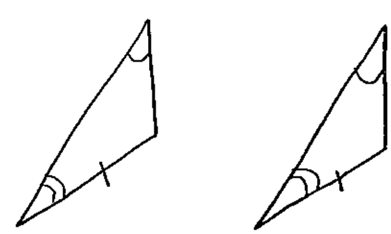
$\angle QRS$

b. Which segment has length equal to NP?

$\overline{SQ}$

Lesson 7-2: Triangle Congruence Theorems

Vocabulary

Congruence Theorem	Picture
SSS Triangle Congruence Theorem: If 3 sides of one Δ are ≅ to 3 sides of another, then the Δ's are ≅.	
SAS Triangle Congruence Theorem: If 2 sides & the angle between is ≅ to 2 sides & the angle between on another, then the Δ's are ≅	
ASA Triangle Congruence Theorem: If 2 angles & the side bet. is ≅ to 2 angles & the side between on another, then the Δ's are ≅.	
AAS Triangle Congruence Theorem: If 2 angles & the side NOT between is ≅ to 2 angles & the side NOT between on another, then the Δ's are ≅.	

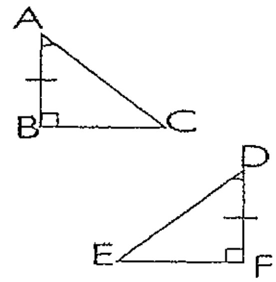
Practice

For #1-4, a) tell whether the triangles are congruent, b) justify with a triangle congruence theorem, c) indicate the corresponding vertices. Otherwise, write "Not enough information."

1. a. Yes

   b. ASA

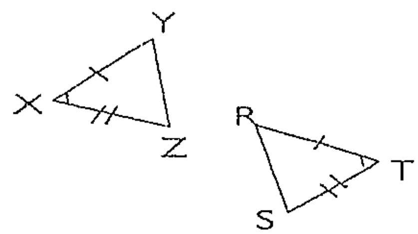
   c. $\triangle ABC \cong \triangle DFE$



2. a. Yes

   b. SAS

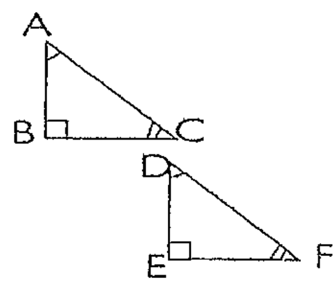
   c. $\triangle XYZ \cong \triangle RTS$



3. a. NO

   b. N.E.I.

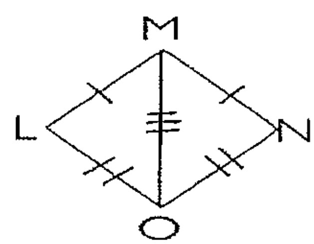
   c. ↓



4. a. Yes

   b. SSS

   c. $\triangle MLO \cong MNO$

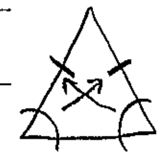


Lesson 7-3: Proofs Using Triangle Congruence

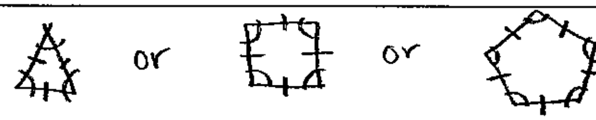
Vocabulary

Justification: a theorem, postulate, or definition that enables a conclusion drawn.

Isosceles Triangle Bases Angles Converse Theorem (ITBAT Converse): If two angles of a $\triangle$ are $\cong$, then the two sides opposite them are also $\cong$.

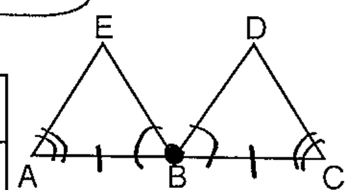


Regular Figure: a figure w/ all sides =, and all angles =.

ex: 

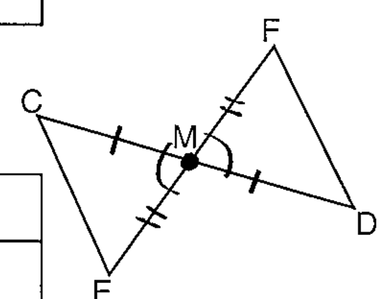
Practice

1. Given: In the figure below, $\angle EBA \cong \angle DBC$, B is the midpoint of AC, and $\angle A \cong \angle C$
Prove: $\triangle ABE \cong \triangle CBD$.



Conclusions	Justifications
0. $\angle EBA \cong \angle DBC$, B is the midpoint of AC, and $\angle A \cong \angle C$	Given
1. $AB \cong BC$	Def. of midpoint
2. $\triangle ABE \cong \triangle CBD$	ASA

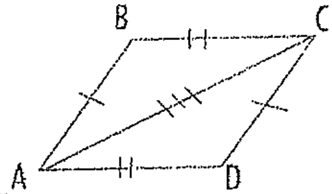
2. Given: M is the midpoint of CD and EF
Prove: $\triangle CME \cong \triangle DMF$



Conclusions	Justifications
0. M is midpoint of CD & EF	Given
1. CM = MD & EM = MF	Definition of midpoint
2. $\angle CME = \angle FMD$	Vertical Angle Theorem
3. $\triangle CME \cong \triangle DMF$	SAS Triangle Congruence

3. Given: AB ≅ CD, BC ≅ AD.

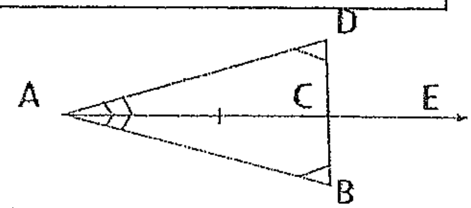
Prove: ∠B ≅ ∠D.



Conclusions	Justifications
0. AB ≅ CD, BC ≅ AD	Given
1. AC ≅ AC	Reflexive Prop.
2. ΔABC ≅ ΔCDA	SSS
3. ∠B ≅ ∠D	CPCF

4. Given: AE bisects ∠BAD and ∠B ≅ ∠D.

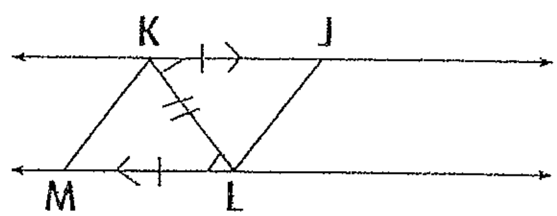
Prove: ΔABC ≅ ΔADC.



Conclusions	Justifications
0. AE bisects ∠BAD & ∠B ≅ ∠D	Given
1. ∠BAC ≅ ∠DAC	Definition of bisector
2. AC ≅ AC	Reflexive Prop.
3. ΔABC ≅ ΔADC	AAS

5. Given: KJ // ML and KJ ≅ LM.

Prove: ΔKJL ≅ ΔLMK.



Conclusions	Justifications
0. KJ // ML & KJ ≅ LM	Given
1. ∠JKL ≅ ∠MLK	// Lines → AIA ≅ Theorem
2. KL ≅ KL	Reflexive Property of Congruence
3. ΔMKL ≅ ΔJKL	SAS Δ ≅ Theorem

Lesson 7-4: Overlapping Triangles

Vocabulary

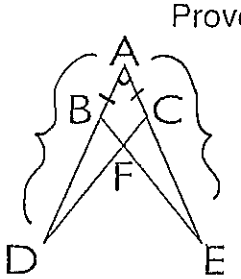
Overlapping Figures: figures that have some parts of their interiors in common.

Example

Non-overlapping Figures: figures that do NOT share any part of their interiors.

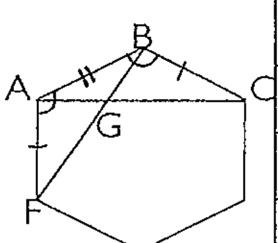
Practice

1. Given: AC = AB and AD = AE.
Prove: ∠D ≅ ∠E.



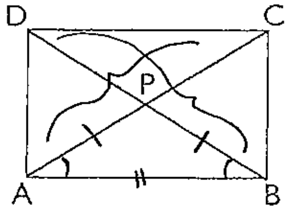
Conclusions	Justifications
0. AC = AB and AD = AE	Given
1. ∠CAD ≅ ∠BAE	Reflexive Prop.
2. ΔADC ≅ ΔAEB	SAS
3. ∠D ≅ ∠E	CPCF

2. Given: Regular hexagon ABCDEF.
Prove: BF ≅ AC.



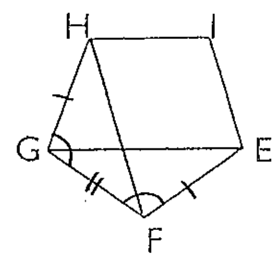
Conclusions	Justifications
0. Regular hexagon ABCDEF	Given
1. AF ≅ BC	Def. of regular
2. ∠FAB ≅ ∠ABC	Def. of regular
3. AB ≅ AB	Reflexive Prop.
4. ΔFAB ≅ ΔCBA	SAS ≅ Theorem
5. BF ≅ AC	CPFC

3. Given: PA ≅ PB, CA ≅ DB.
Prove: DA ≅ CB.



Conclusions	Justifications
0. PA ≅ PB & CA ≅ DB	Given
1. BA ≅ AB	Reflexive Prop.
2. ∠PAB ≅ ∠ABP	Isosceles Triangle Base Angles Theorem (ITBAT)
3. ΔDAB ≅ ΔCBA	SAS Δ ≅ Theorem
4. DA ≅ CB	CPCF

4. Given: Regular pentagon EFGHI.
Prove: EG ≅ FH.



Conclusions	Justifications
0. Regular pentagon EFGHI	Given
1. EF ≅ GH	Def. of regular
2. ∠EFG ≅ ∠FGH	Def. of regular
3. FG ≅ FG	Reflexive Prop.
4. ΔEFG ≅ ΔFGH	SAS
5. EG ≅ FH.	CPCF

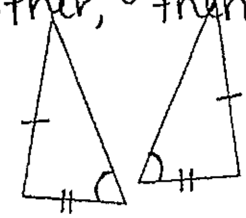
Lesson 7-5: SsA & HL Congruence

Ever wondered why we never use SSA? Well, we will start to use it here...but we never actually call it SSA, this is because it only works in specific cases. The two specific cases are SsA and HL.

Vocabulary

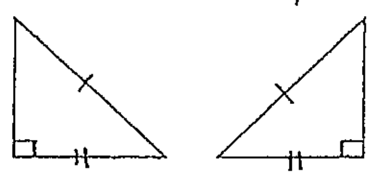
SsA Congruence: If a longer side & angle w/ a short side between are ≅ to a longer side & angle w/ a short side between on another, then the △'s are ≅.

Example



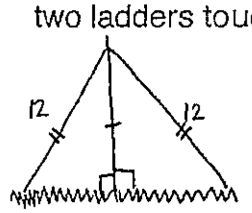
HL Congruence: If a hypotenuse & leg are ≅ to a hypotenuse & leg of another, then the △'s are ≅.

Example

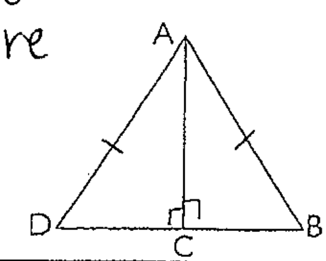


Practice

1. A high fence is perpendicular to the ground. Jack leans a 12' ladder against the fence. On the other side of the fence, Jill also leans a 12' ladder against the fence, reaching the same height as the first ladder. Why is it no surprise that the two ladders touch the ground at the same distance from the fence?



By HL the △'s are ≅, so by CPCF the other legs are also ≅.

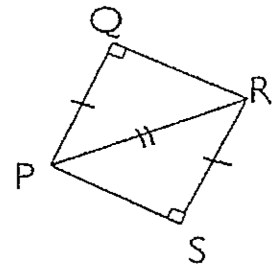


2. Given: $AC \perp BD$ and $AD = AB$.

   Prove: $\triangle ABC \cong \triangle ADC$.

Conclusions	Justifications
0. $AC \perp BD$ & $AD = AB$	Given
1. $AC \cong AC$	Reflexive Prop.
2. $\triangle ABC \cong \triangle ADC$	HL

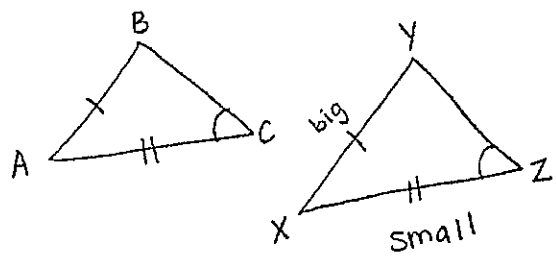
3. Given: PQ ≅ RS, ∠Q and ∠S are right angles.
Prove: ∠QPR ≅ ∠SRP



Conclusions	Justifications
0. PQ ≅ RS, ∠Q = ∠S = 90°	Given
1. PR ≅ PR	Reflexive Prop.
2. ΔPQR ≅ ΔRSP	HL
3. ∠QPR ≅ ∠SRP	CPCF

4. Given: $\overline{AB} \cong \overline{XY}$, $\overline{AC} \cong \overline{XZ}$, $\angle C \cong \angle Z$ and $XY > XZ$.

Are ΔABC and ΔXYZ congruent? Justify with a congruence statement and indicate the corresponding vertices.



Yes, by SsA.

ΔABC ≅ ΔXYZ.