Name: KEY!

Hour: ______

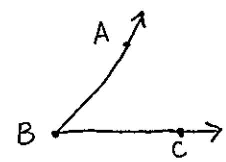
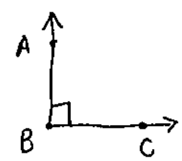
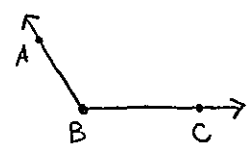
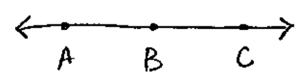
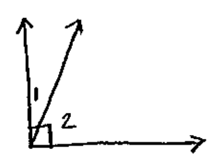
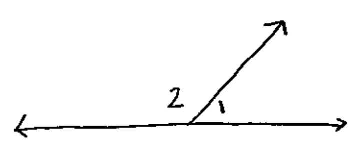
Unit B: Angles

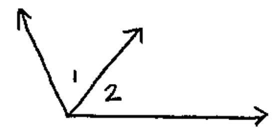
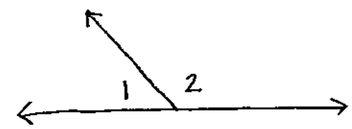
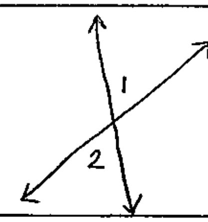
Proof that 1 = 2:

- *Assume* $a = b$
- $a * b = b^2$
- $a * b - a^2 = b^2 - a^2$
- $a*(b-a) = (b+a)*(b-a)$
- $a = b+a$
- $a = 2a$ *(because* $a=b$*)*
- *Therefore,* $1 = 2$ *!!*

Lesson 3-3: Properties of Angles

Vocabulary

Angle Type	Definition	Picture
Zero	Angle w/ measure of 0°	
Acute	between 0° & 90°	
Right	90°	
Obtuse	between 90° & 180°	
Straight	180°	
Complimentary Angles	add up to 90°	
Supplementary Angles	add up to 180°	

Angle Type	Definition	Picture
Adjacent Angles	share a side	
Linear Pair	share a side & add up to 180°.	
Vertical Angles	opposite each other & congruent	

Practice

1. An angle is ¼ the measure of its supplement. Find the measure of the angle and check your answer.

add to 180

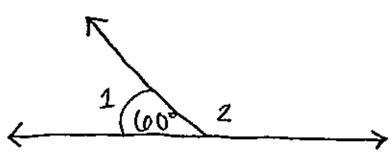
$$\frac{1}{4}x + x = 180$$

$$\frac{1.25x}{1.25} = \frac{180}{1.25}$$

$$x = 144 \div 4 \quad \boxed{= 36^\circ}$$

2. Suppose ∠1 and ∠2 are supplementary and adjacent angles, with m∠1 = 60.

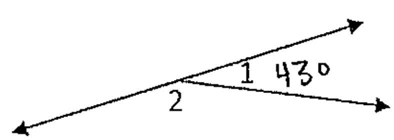
a. Sketch a possible diagram.



b. Find m∠2.

$$60 + \angle 2 = 180$$
$$-60 \qquad -60$$
$$\boxed{\angle 2 = 120^\circ}$$

3. Refer to the diagram to the right.



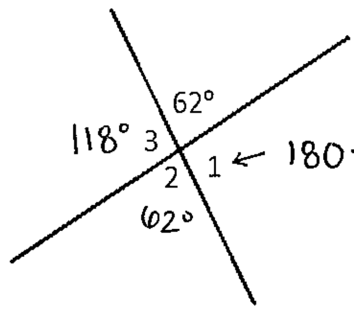
a. If m∠1 = 43, what is m∠2?

$$180 - 43 = \boxed{137^\circ}$$

b. If m∠1 = x, what is m∠2?

$$x + \angle 2 = 180$$
$$-x \qquad -x$$
$$\boxed{\angle 2 = 180 - x}$$

4. Find the measures of as many angles as you can in the figure below, given that the labeled angle is 62 degrees.



$\leftarrow 180-62=118°$

$\angle 1 = 118°$

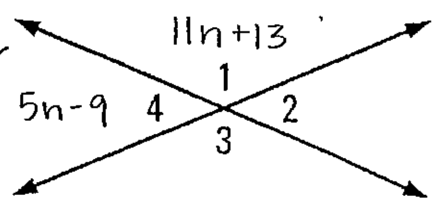
$\angle 2 = 62°$

$\angle 3 = 118°$

5. Suppose $m\angle 1 = 11n + 13$ and $m\angle 4 = 5n - 9$.

linear pair/supplementary

Find $m\angle 3$.



$11n + 13 + 5n - 9 = 180$

$16n + \cancel{4} = 180$

$\quad\;\; \cancel{-4} \quad -4$

$\frac{\cancel{16}n}{\cancel{16}} = \frac{176}{16}$

$n = 11$

So $\angle 1 = 11n + 13$

$\angle 1 = 11(11) + 13$

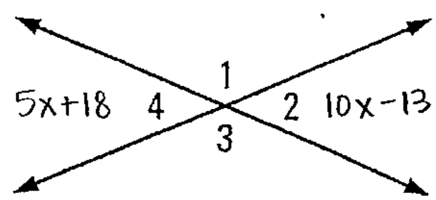
$\angle 1 = 134°$, so does $\angle 3$!

$\boxed{\angle 3 = 134°}$

6. Suppose $m\angle 4 = 5x + 18$ and $m\angle 2 = 10x - 13$.

vertical angles!

Find $m\angle 4$.



$\cancel{5x} + 18 = 10x - 13$

$\cancel{-5x} \qquad\quad -5x$

$18 = 5x \cancel{-13}$

$+13 \qquad \cancel{+13}$

$\frac{31}{5} = \frac{\cancel{5}x}{\cancel{5}}$

$6.2 = x$

So, $\angle 4 = 5x + 18$

$\angle 4 = 5(6.2) + 18$

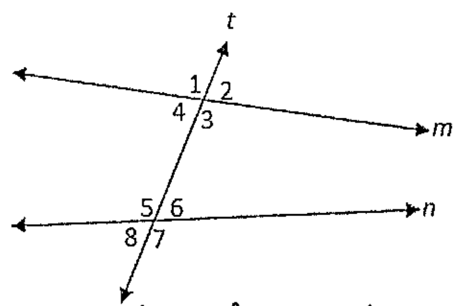
$\boxed{\angle 4 = 49°}$

Lesson 3-6: Parallel Lines

Vocabulary

Transversal: a line that "cuts" or intersects two lines.

Example



t is the transversal

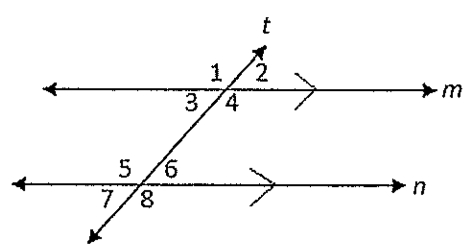
Corresponding Angles: any pair of angles in similar locations w/ respect to the transversal & each line.

Corresponding Angles Postulate: Suppose two coplanar lines are cut by a transversal.

a. If two corresponding angles have the same measure, then the lines are parallel.

b. If the lines are parallel, then the corresponding angles have the same measure.

Example



$\angle 1$ & $\angle 5$

$\angle 2$ & $\angle 6$

$\angle 4$ & $\angle 8$

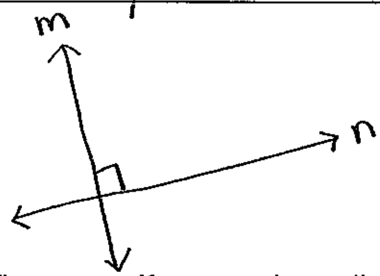
$\angle 3$ & $\angle 7$

Lesson 3-7: Perpendicular Lines

Vocabulary

$\perp$ Perpendicular: two segments, rays, or lines are $\perp$ if & only if they form a $90°$ angle.

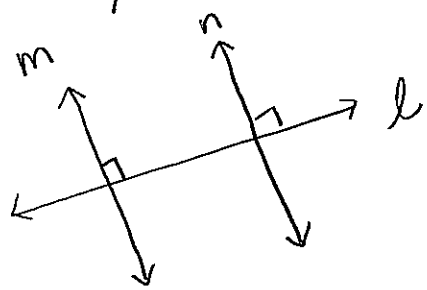
Example



$m \perp n$

Two Perpendiculars Theorem: If two coplanar lines *m* and *n* are each perpendicular to the same line *l*, then they are // to each other.

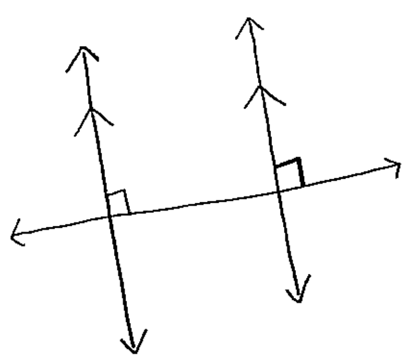
Example



$m // n$

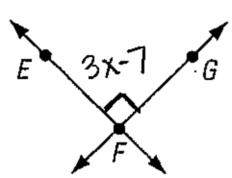
Perpendicular to Parallels Theorem: In a plane, if a line is perpendicular to one of two parallel lines, then it is also $\perp$ to the other line.

Example



Practice

1. In the figure below, the two lines are perpendicular and m∠EFG = 3x - 7. Find *x*.



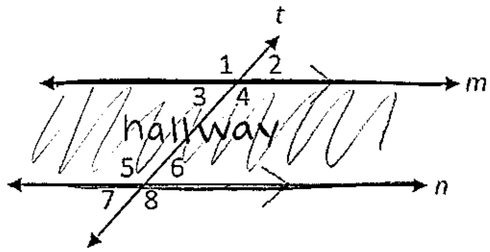
$3x \cancel{-7} = 90$

$\cancel{+7} \quad +7$

$\frac{\cancel{3}x}{\cancel{3}} = \frac{97}{3}$

$\boxed{x = 32.\overline{3}}$

Lesson 5-4: Proofs Using Transitivity



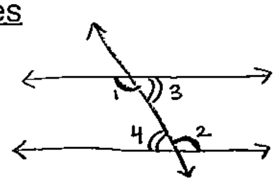
Interior Angles: $\angle 3, \angle 4, \angle 5, \angle 6$

Exterior Angles: $\angle 1, \angle 2, \angle 7, \angle 8$

Vocabulary

≅ AIA: angles on opposite sides of a transversal & on the interior of the // lines

Examples

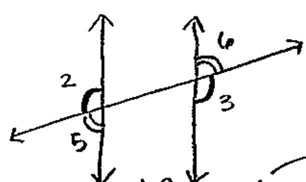


$\angle 1 = \angle 2$

$\angle 3 = \angle 4$

≅ AEA: angles on opposite sides of a transversal & on the exterior of the // lines

Examples



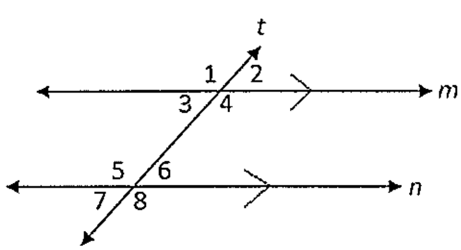
$\angle 2 = \angle 3$

$\angle 5 = \angle 6$

Transitive Property: If A = B and B = C, then you know A = C as well. ("lets you jump")

WHY are alternate interior angles congruent and alternate exterior angles congruent?

AIA



Check it out... $\angle 3 = \angle 6$ since...

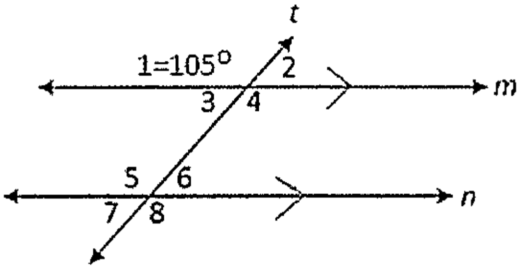
① $\angle 3 = \angle 2$ they are vertical angles

② $\angle 2 = \angle 6$ they are corresponding angles

③ So, $\angle 3 = \angle 6$ by the Transitive Property

Practice

1. If ∠1 = 105, find all the other missing angles.



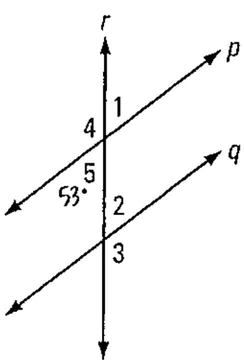
∠2 = 75° ∠3 = 75°

∠4 = 105° ∠5 = 105°

∠6 = 75° ∠7 = 75°

∠8 = 105°

2. Use the figure below in which p // q. If $\underbrace{m\angle 5 = 5j - 27 \text{ and } m\angle 2 = 3j}_{\text{AIA}} + 5$, find the measure of each angle.



$$5j - 27 = \cancel{3j} + 5$$
$$-3j \qquad \cancel{-3j}$$

$$2j \cancel{-27} = 5$$
$$\cancel{+27} \quad +27$$

$$\frac{2j}{\cancel{2}} = \frac{32}{2}$$
$$j = 16$$

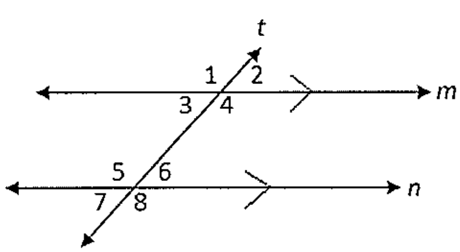
So, ∠5 = 5j − 27
∠5 = 5(16) − 27
∠5 = 53°

∠1 = 53°

∠3 = 127°

∠4 = 127°

3. Explain why ∠2 = ∠7 without using AEA.



∠2 = ∠3 since they are vertical angles.

∠3 = ∠7 since they are corresponding angles.

∠2 = ∠7 by the Transitive Property.

Proofs with Angles

Vocabulary

Proof: a series of reasoning made to show why a statement is true.

Given Statement: a piece of info. you start w/ that you can assume is true.

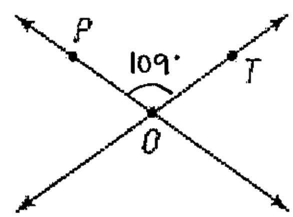
Prove/Conclusion Statement: the piece of info. you intend to show is true.

Justification: "Why" a statement is true (definition, property, postulate, or theorem)

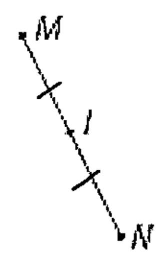
Practice

Write a justification for each conclusion.

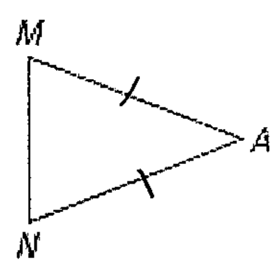
1. Given: $m\angle POT = 109$.
   Conclusion: $\angle POT$ is obtuse.
   def. of obtuse



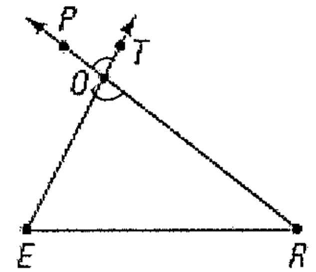
2. Given: I is the midpoint of $\overline{MN}$.
   Conclusion: $MI = IN$
   def. of midpoint



3. Given: $MA = AN$.
   Conclusion: ΔMAN is isosceles.
   def. of isosceles

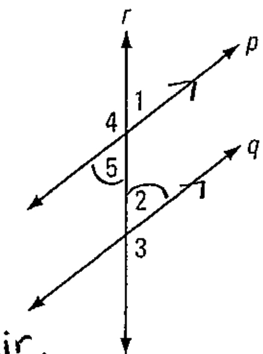


4. Given $\angle POT$ and $\angle ROE$ are vertical angles.
Conclusion: $m\angle POT = m\angle ROE$.



Vertical Angles Theorem

In 5 & 6 Use the diagram at the right below.



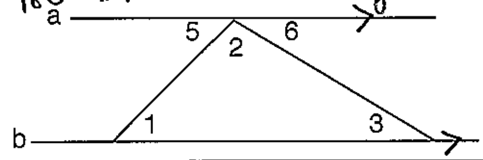
5. Given: $p \parallel q$
Conclusion: $\angle 2 = \angle 5$

AIA

6. Given: $p \parallel q$
Conclusion: $\angle 4$ is supplementary to $\angle 2$.

$\angle 4 + \angle 5 = 180°$ since they are a linear pair,
$\angle 5 = \angle 2$ by AIA, so $\angle 4 + \angle 2 = 180$ by Transitivity.

7. Given: $a \parallel b$
Conclusion: $\angle 1 + \angle 2 + \angle 3 = 180$

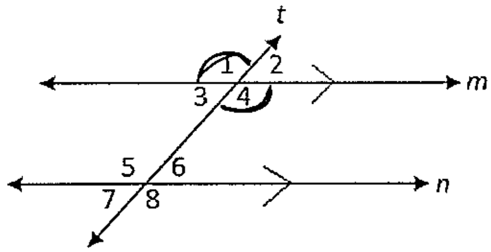


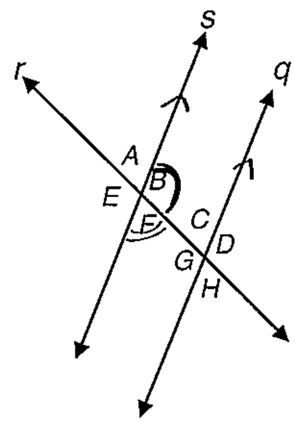
Conclusions	Justifications
0. $a \parallel b$	Given
1. $\angle 5 + \angle 2 + \angle 6 = 180$	Linear Pair / def. of supplementa
2. $\angle 1 = \angle 5$	AIA
3. $\angle 3 = \angle 6$	AIA
4. $\angle 1 + \angle 2 + \angle 3 = 180$	Substitution

So, we just proved that the angles of a $\triangle$ add up to 180° → "Triangle Sum Theorem"

8. Given: m // n, s // q, and ∠1 = ∠B

Prove: ∠4 + ∠F = 180





Conclusions	Justifications
0. m//n, s//q & ∠1=∠B	Given
1. ∠1 = ∠4	Vertical Angles
2. ∠4 = ∠B	Transitive Property
3. ∠B + ∠F = 180	Linear Pair
4. ∠4 + ∠F = 180	Substitution