Name: KEY!

Hour: ______

Unit A: Logic

Geometry 1st Semester

Lesson 2-2: "If-Then" Statements

$$\overbrace{\text{If \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}}^{\text{Proposition}}, \text{ then } \overbrace{\text{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}}^{\text{Implication}}.$$

Antecedent — Consequent

Vocabulary

Conditional: a sentence w/ an "if" clause & a "then" clause; aka "if...then..." statement

Instance: a specific case where the "if" part is TRUE, & the "then" part is TRUE. (T, T)

Counterexample: a specific case where the "if" part is TRUE, & the "then" part is FALSE. (T, F)

$\Rightarrow$: "implies"; $p \Rightarrow q$ means "p implies q"

$\Leftrightarrow$: "if & only if"; $p \Leftrightarrow q$ means "p if & only if q"

Practice

a = It is a rabbit
b = It has four legs
c = It has floppy ears

Write the sentence symbolized by each statement.

1. $a \Rightarrow b$

   If it is a rabbit, then it has 4 legs.

2. If b, then a.

   If it has 4 legs, then it is a rabbit.

3. c implies a $c \Rightarrow a$

   If it has floppy ears, then it is a rabbit.

Write the proposition and implication of the conditional.

4. If a network has four nodes, then it has six arcs.

   Proposition: If a network has 4 nodes

   Implication: then it has 6 arcs

Rewrite the following statements as conditionals.

5. A person that is 14 years old is a teenager.

   If a person is 14 yrs. old, then they are a teenager.

6. A Doberman is a dog.

   If an animal is a Doberman, then it is a dog.

Given the conditional, "If $c \geq 3$, then $c < 10$."

7. Give an instance of the conditional.

   T, T

   $c = 5$

8. Give a counterexample to the conditional.

   T, F

   $c = 15$

Lesson 2-3: Converses & Biconditionals

Vocabulary

Converse: the converse of $p \Rightarrow q$ is $q \Rightarrow p$.
*just b/c its opposite... doesn't tell us whether its T or F.

Biconditional ($\Leftrightarrow$): "if & only if"; $p \Leftrightarrow q$ means "p if & only if q". Its the combination of 2 conditionals, $p \Rightarrow q$ & $q \Rightarrow p$.

Practice

1. If you are in Grand Rapids, then you are in Michigan.

   Converse: If you are in MI, then you are in G.R.

2. If you have a Doberman, then you have a dog.

   Converse: If you have a dog, then you have a Doberman.

3. If $x > 1$, then $x \geq -2$.

   Converse: If $x \geq -2$, then $x > 1$.

4. If $x = 2$, then $3x + 1 = 7$.

   Converse: If $3x + 1 = 7$, then $x = 2$.

5. If a person is driving 100mph on a U.S. highway, then the person is speeding.

   a. Write the converse of the conditional.
   If a person is speeding, then they are driving 100 mph on a U.S. highway.

   b. Is the original statement true? Is the converse true?
   Original: true
   Converse: false, could be going 80mph

6. Let p be the statement $x < 5$. Let q be the statement $x < 4$.

a. Write $p \rightarrow q$.

If $x < 5$, then $x < 4$.

b. Is $p \rightarrow q$ true? Explain your answer.

No, let $x = 4.5$

c. Write the converse of the statement $p \rightarrow q$.

If $x < 4$, then $x < 5$.

d. Is the converse true?

Yes

7. Let p = "A country is democratic". Let q = "The power resides in the people".

Write $p \Leftrightarrow q$ in words.

A country is democratic if & only if the power resides in the people.

8. Given the statement: "A right angle is an angle whose measure is 90."

a. Write a conditional (if-then statement) for this statement.

If an angle is right, then its measure is 90.

b. Write the converse of your statement in part a.

If an angle measures 90, then it is right.

c. Are both *a* and *b* true? If so, write the definition of a right angle as a biconditional.

Yes.

An angle is right if & only if its measure is 90.

Lesson 11-2: Negations

Vocabulary

Negation: the negation of a statement p is called "not p"; symbol used: $\sim$ or $\neg$ (if p is T, $\sim p$ is F)

Inverse: the inverse of $p \to q$ is $\sim p \to \sim q$.
logically equivalent to the converse ($q \to p$)

Contrapositive: the contrapositive of $p \to q$ is $\sim q \to \sim p$.
logically equivalent to the conditional ($p \to q$).

Practice

Orig. $p \to q$
Conv. $q \to p$
Inv. $\sim p \to \sim q$
contrapos. $\sim q \to \sim p$

1. $\underbrace{\text{If you live in California,}}_{p}$ $\underbrace{\text{then you need a mountain bike.}}_{q}$

$q \to p$ Converse: If you need a mountain bike, then you live in CA.

$\sim p \to \sim q$ Inverse: If you don't live in CA, then you don't need a mountain bike.

$\sim q \to \sim p$ Contrapositive: If you don't need a mountain bike, then you don't live in CA.

2. $\underbrace{\text{If you live in an air-conditioned home,}}_{p}$ $\underbrace{\text{then you have the opportunity to be cool in the summer.}}_{q}$

$q \to p$ Converse: If you have the opportunity to be cool in the summer, then you live in A.C. home.

$\sim p \to \sim q$ Inverse: If you don't live in A.C. home, then you don't have the opportunity to be cool in the summer.

$\sim q \to \sim p$ Contrapositive: If you don't have the opportunity to be cool in the summer, then you don't live in A.C. home.

3. Write your own "If...then" Statement:

If you own a Yaris, then it is a Toyota.

$q \to p$ Converse: If you own a Toyota, then it is a Yaris.

$\sim p \to \sim q$ Inverse: If you don't own a Yaris, then it's not a Toyota.

$\sim q \to \sim p$ Contrapositive: If you don't own a Toyota, then its not a Yaris.

4. Make a conclusion from these two statements.
   (a) Riley cannot become an eagle scout.
   (b) If a person is a boy scout, he can become an eagle scout.

Riley is not a boy scout.

Lesson 11-1: Logic of Making Conclusions

Vocabulary

Law of Detachment: given a statement $p \to q$ & statement p, you can conclude q.

Law of Transitivity: given a statement $p \to q$ & $q \to r$, you can conclude $p \to r$.

Law of Contrapositive: given a statement $p \to q$ & statement $\sim q$, you can conclude $\sim p$.

Law	Symbols	Example
Law of Detachment	Given: (1) $p \to q$ (2) p conclude: q	(1) If $x = 10$, then $y = 6$. (2) $x = 10$. Conclude: $y = 6$
Law of Transitivity	Given: (1) $p \to q$ (2) $q \to r$ conclude: $p \to r$	(1) If $x = 10$, then $y = 6$. (2) If $y = 6$, then $z = 21$. Conclude: If $x = 10$, then $z = 21$.
Law of Contrapositive	Given: (1) $p \to q$ (2) $\sim q$ conclude: $\sim p$	(1) If $x = 10$, then $y = 6$. (2) $y = 3$. Conclude: $x \neq 10$.

*any other combinations, we can't make a conclusion, so write "not enough information"

Practice

1. A commercial states: *If you want to be popular, you must dress well.*
   If you want to dress well, you wear Brand X jeans.
   What conclusion(s) can you make (if any)?

   If you want to be popular, you wear Brand X jeans.

2. (1) Every rhombus is a kite.
   (2) The diagonals of a kite are perpendicular.
   (3) MBUS is a rhombus.

   What conclusion(s) can you make (if any)?

   MBUS is a kite & the diagonals are $\perp$.

3. (1) Some bracelets are valuable jewelry.
   (2) All bracelets are made of gold.

   What conclusion(s) can you make (if any)?

   Some gold bracelets are valuable.

4. (1) If you own a Doberman, then you own a dog.
   (2) You own a dog.

   What conclusion(s) can you make (if any)?

   no conclusion

5. (1) My gardener is well worth listening to on military subjects.
   (2) No one can remember the battle of Waterloo, unless he is very old.
   (3) Nobody is really worth listening to on military subjects, unless he can remember the battle of Waterloo.

   What conclusion(s) can you make (if any)?

   My gardener is old.

6. (1) If $a = 2$, then $b = 17$.
   (2) $b \neq 17$.

   What conclusion(s) can you make (if any)?

   $a \neq 2$.

Lesson 11-4: Indirect Proofs

Vocabulary

Direct Reasoning: begins with information known to be true

Direct Proofs: assume the given information is true & prove it.

Indirect Reasoning: assume the given statement is true & show a contradiction

1. If you want to prove a statement to be false, start by reasoning from it.
   Example: Prosecutors thought the defendant was guilty, the lawyer reasoned from this.

2. Using valid logic, try to make the reasoning lead to a contradiction or other false statements.
   Example: The lawyer argued that the defendant would have been in two places at once.

3. If the reasoning leads to a contradiction or other false statements, the assumed statement must be false.
   Example: The lawyer concluded that the defendant was not guilty.

Contradictory: two statements are contradictory if they both cannot be true at the same time.

Law of Indirect Reasoning: If a valid reasoning from a statement p leads to a false conclusion, then p is false.

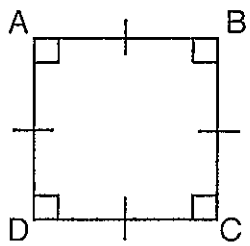
Practice

1. Let *p* be the statement "∠V is acute." Let statement *q* be the statement "∠V is right." Are *p* and *q* contradictory? Explain your answer.

   Yes, acute is less than 90, right is equal to 90. An angle can't be both acute & right at the same time.

2. In the figure below, let p = ABCD is a rhombus. Let q = ABCD is a rectangle. Are p and q contradictory? Explain your answer.

No, p & q can both be true at the same time.
ABCD is both a rhombus & a square.



3. Show that $3(4 + 2x) = 6(x + 1)$ is never true.

$$12 + 6x = 6x + 6$$
$$-6x \qquad -6x$$
$$12 = 6$$

Since $12 = 6$ is a false conclusion, the original statement is false for all values of x.

* 4. Write an indirect proof argument to show that $\sqrt{22{,}200} \neq 149$.

Given: The real numbers $\sqrt{22{,}200}$ and 149.

Prove: $\sqrt{22{,}200} \neq 149$

1. $\sqrt{22{,}200} = 149$

2. $\left(\sqrt{22{,}200}\right)^2 = (149)^2$

$$22{,}200 = 22{,}100$$

3. Since $22{,}200 \neq 22{,}100$ we know $\sqrt{22{,}200} \neq 149$.