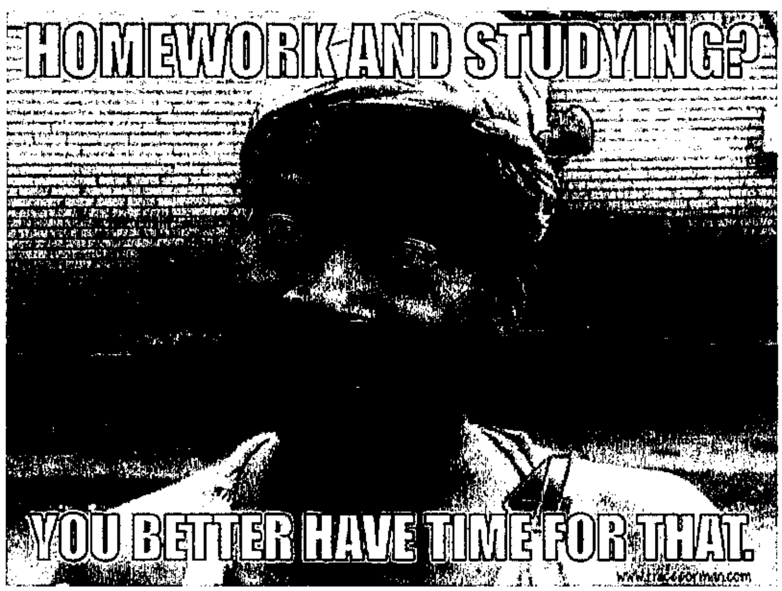
Name: KEY!

Hour: ________

Chapter 9:

EXPONENTIAL &

Logarithm FUNCTIONS



Lesson 9-1: Exponential Growth

Vocabulary

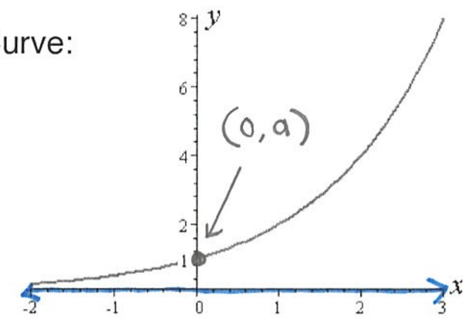
Exponential Growth: growth by multiplication (rather than addition). Quantities that grow "exponentially" grow much faster!

Exponential Model:

$$y = ab^x$$

x ← growth period
b ← growth factor (pos. #)
a ← initial value (positive #)

Exponential Curve:



X's
Domain: all real #'s

Y's
Range: positive real #'s

Asymptote: a line that a graph approaches, but NEVER touches.

Growth Factors

Percent Increase: an increase in addition to the existing amount (100%)

Examples:	% Increase		Growth Factor
	22%	→ 100% + 22% = 122% =	$\boxed{1.22}$
	50%	→ 100% + 50% = 150% =	$\boxed{1.5}$
	3.8%	→ 100% + 3.8% = 103.8% =	$\boxed{1.038}$

Practice

1. The Consumer Price Index (CPI) measures the costs of goods and services in the US. In 1980, the CPI in the US was \$100. Between 1980 and 1990 the CPI rose at an average rate of 4.7% per year.

   initial value

   growth rate: 1.047

   a. Let x be the number of years after 1980 and y be the CPI. Write an exponential model to represent the situation.

   $$y = ab^x \rightarrow \boxed{y = 100(1.047)^x}$$

   b. Graph the function using the window ($-10 \le x \le 25$, $0 \le y \le 300$).

   c. Find the CPI in 1985. ← 5 yrs after 1980

   $$y = 100(1.047)^5 \rightarrow \$125.82$$

   d. What year will the CPI be approximately \$250?

   ≈ 20 yrs later, so $\boxed{2000}$

2. The world population in 1985 was 4.9billion people. It was estimated that the population would double every 35 years.

   initial value

   growth rate = 2

   a. Write an exponential equation modeling this situation. Let x represent the years since ~~1985, and let~~ p represent the total estimated population in billions.

   $$y = ab^x$$

   $$\boxed{p = 4.9(2)^{x/35}}$$

   b. Using the model, what would be an estimate for the world's population in the year 2000?

   15yrs later

   $$p = 4.9(2)^{15/35}$$

   $$\boxed{p = 6.59 \text{ billion}}$$

   c. Using this model, what would be an estimate for the world's population in the year 1980?

   5yrs ago → -5

   $$p = 4.9(2)^{-5/35}$$

   $$\boxed{p = 4.44 \text{ billion}}$$

Lesson 9-2: Exponential Decay

Vocabulary

Exponential Decay: occurs when the growth factor is less than 1 (but still larger than 0)

$$y = ab^x$$

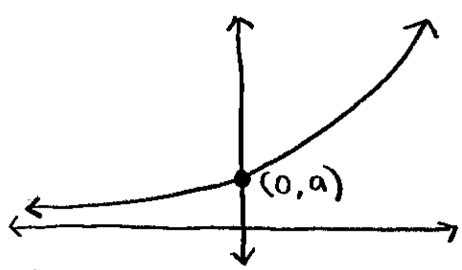
growth period
growth factor (bet. 0 & 1)
initial value (pos. #)

Depreciation: when value decreases over time

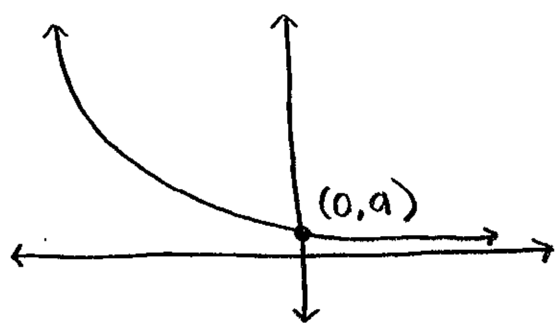
Half-Life: the amount of time it takes for half of the material to decay.

Graphs of Exponential Models

Growth Factor B > 1



Growth Factor 0 < B < 1



Growth Factors

Exponential Decay: Starting w/ 100% & taking away a percentage.

Examples:	% Decrease		Growth Factor
	12%	100% - 12% =	88% = $\boxed{.88}$
	51%	100% - 51% =	49% = $\boxed{.49}$

Practice

1. Some used-car dealers use the general rule-of-thumb that the trade value of a car decreases by 30% each year.

   a. If a car's value decreases by 30%, what value does it retain?

   $100\% - 30\% = 70\%$ or $\boxed{.7}$

   b. Ethan has a car worth $6400. Write an equation that models the value of Ethan's car in x years.

   ← initial value

   growth factor

   $\boxed{y = 6400(.7)^x}$

   c. How much will Ethan's car be worth in 3 years?

   ↳ growth period

   $y = 6400(.7)^3$

   $\approx \boxed{\$2{,}195.20}$

   d. Ethan has owned the car for 2 years. How much was it worth when he bought it?

   ↳ growth period = −2

   $y = 6400(.7)^{-2}$

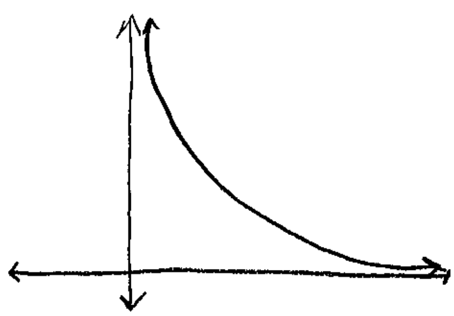
   $\boxed{\approx \$13{,}061.22}$

2. An altitude of less than 80km, standard atmospheric pressure (1035g/cm²) is halved for each 5.8km of vertical descent.

   a. Describe the relationship between altitude x and pressure y.

   $y = 1035(.5)^{x/5.8}$

   b. Sketch a graph of the equation using window ($-1 \le x \le 60$, $-2 \le y \le 1050$).



   c. Find the atmospheric pressure at an altitude of 40km.

   $y = 1035(.5)^{40/5.8}$

   $\boxed{y \approx 8.69 \text{ g/cm}^2}$

Lesson 9-3: Continuous Growth

Vocabulary

e: a constant # used in many mathematical situations (an irrational #) ; e=2.71828182845..

- Named after Leonhard Euler
- John Napier first used "e" (he just didn't know it yet!)
- John Napier was attempting to make a model that would relate multiplication to addition.
- The discovery of "e" lead to the birth of modern mathematics!

Continuous Growth: occurs when exponential growth is being compounded constantly.

- As the compound frequency increases, the function is equal to an irrational number.
- Continuous Growth & Decay are used for many growth models found in nature.
- It is possible (but rare) for bank account to offer continuous interest.

Continuous Interest Formula	Continuous Growth Model
If interest is being compounded continuously, then... int. rate $A = Pe^{rt}$ ← time start amt	For any function that grows continuously... $N(t) = N_0 e^{rt}$

Practice

start amt   r = .05

1. If \$850 is invested at an interest rate of 5% compounded continuously, how much will be in the account after 10 years?

t

$$A = Pe^{rt}$$

$$A = 850 \cdot e^{.05 \cdot 10}$$

$$A = \boxed{\$1401.41}$$

r = -.25

2. A machine used in industry depreciates continuously at a rate of 25% per year. Currently the machine is worth \$28,000. Write a model to represent the machine's value after *t* years.

start amt.

$$A = 28{,}000\, e^{-.25t}$$

How much is the machine worth in 3 years?

t

$$A = 28{,}000\, e^{-.25 \cdot 3}$$

$$= \boxed{\$13{,}226.26}$$

How much was the machine worth 2 years ago?

t = -2

$$A = 28{,}000\, e^{-.25 \cdot -2}$$

$$A = \boxed{\$46{,}164.20}$$

Logarithms Day #1: The Basics

Vocabulary

Logarithm: the inverse of the exponential function

Used for: solving for a missing exponent

Equation: $\log_b a = x$

base equals exponent

SAME!

$b^x = a$

Practice

Rewrite each equation in exponential form.

1) $\log_6 36 = 2$

$6^2 = 36$

2) $\log_{289} 17 = \frac{1}{2}$

$289^{1/2} = 17$

3) $\log_{14} \frac{1}{196} = -2$

$14^{-2} = \frac{1}{196}$

4) $\log_3 81 = 4$

$3^4 = 81$

Rewrite each equation in logarithmic form.

5) $64^{\frac{1}{2}} = 8$

$\log_{64} 8 = \frac{1}{2}$

6) $12^2 = 144$

$\log_{12} 144 = 2$

7) $9^{-2} = \frac{1}{81}$

$\log_9 \frac{1}{81} = -2$

8) $\left(\frac{1}{12}\right)^2 = \frac{1}{144}$

$\log_{\frac{1}{12}} \frac{1}{144} = 2$

Evaluate each expression.

9) $\log_4 64 = x$

$4^x = 64$

so, $\boxed{x = 3}$

10) $\log_6 216 = x$

$6^x = 216$

so, $\boxed{x = 3}$

Logarithms Day #2: Properties of Logs

Vocabulary

Logarithm: the inverse of exponential form

Logarithmic Form		Exponential Form
$\log_b a = x$	$\longrightarrow$	$b^x = a$

Common Log: a log w/ a base of 10

*If a log is written without a base, it is automatically considered to be a common log.

* "LOG" button on calc ← only use if base is 10 though!

Natural Log: a log w/ a base of "e"

*On calculator, the button "LN"

Properties of Logs	
Multiplication Property	$\log_b (x \cdot y) \longrightarrow \log_b x + \log_b y$
Division Property	$\log_b \left(\frac{x}{y}\right) \longrightarrow \log_b x - \log_b y$
Power Property	$\log_b x^y \longrightarrow y \cdot \log_b x$

Practice

1. In your calculator, estimate each to the nearest hundredth. 2 decimals

   a. $\log 100 = 2$ b. $\log 42 = 1.62$ c. $\log .001 = -3$

   d. $\ln 16 = 2.77$ e. $\ln 402 = 6$ f. $\ln 1 = 0$

Simplify (write as a single log).

2. $\log_5 3 + \log_5 x$

$\boxed{\log_5 (3 \cdot x)}$

3. $4\log_2 3 - \log_2 3$

$\log_2 3^4 - \log_2 3$

$\log_2 81 - \log_2 3 \rightarrow \log_2 \left(\frac{81}{3}\right)$

$\boxed{\log_2 27}$

4. $3\log x + 2\log\ y - \log z$

$\log x^3 + \log y^2 - \log z$

$\log(x^3 \cdot y^2) - \log z \rightarrow \boxed{\log \frac{x^3 y^2}{z}}$

5. $4\ln x + 5\ln 3$

$\ln x^4 + \ln 3^5$

$\ln(x^4 \cdot 3^5) = \boxed{\ln(243x^4)}$

Expand (write as multiple logs).

6. $\log_4 5m$

$\boxed{\log_4 5 + \log_4 m}$

7. $\log_2 16x^2$

$\boxed{\log_2 16 + \log_2 x^2}$

8. $\ln\left(\frac{3x^4y^3}{z^2}\right)$

$\boxed{\ln 3x^4y^3 - \ln z^2}$

9. $\log\left(\frac{x}{3}\right)^2 = \log\left(\frac{x^2}{9}\right)$

$\boxed{\log x^2 - \log 9}$

Tricky ones...

10. $\log_5 5 = x$

$5^x = 5$, so $\boxed{x=1}$

11. $\ln e = x$

$e^x = e$ so, $\boxed{x=1}$

12. $\log_b 1 = x$

$b^x = 1$ so, $\boxed{x=0}$

13. $\ln 1 = x$

$e^x = 1$ so, $\boxed{x=0}$

14. $\log_b 0 = x$

$b^x = 0$ $\boxed{\text{no solution}}$

15. $\log(-2) = x$

$\leftarrow 10$

$10^x = -2$ $\boxed{\text{no solution}}$

Logarithms Day #3: Change of Base Formula

Introduction

Solve $\log_5 12 = x$ using a calculator...

↓

only does "common logs"

Oh wait...our calculators can't do base 5, they only do base 10.

So, instead...try writing it in exponential form...

$$5^x = 12$$

It means, 5 to the what power is 12. Can you figure that out using a calculator? In your head?

no...

Check this out!

Start with exponential form... $5^x = 12$

Take a common log of each side... $\log 5^x = \log 12$

base 10

Move the *x* (exponent) down in front... $x \log 5 = \log 12$

Divide to get *x* alone... $\frac{x \cdot \cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 12}{\log 5}$ type in calc!

$$x = \boxed{1.54}$$

So, $5^{1.54} = 12$ & $\log_5 12 = 1.54$

Vocabulary

Properties of Logs	
Change of Base Property	$\log_b a = \dfrac{\log a}{\log b}$

Practice

1. $\log_7 25$

$$\frac{\log 25}{\log 7} = \boxed{1.65}$$

2. $\log_2 .00014$

$$\frac{\log .00014}{\log 2} = \boxed{-12.8}$$

3. $\log_4 5000$

$$\frac{\log 5000}{\log 4} = \boxed{6.14}$$

Logarithms Day #4: Solving Equations w/ Logs

General Guidelines

- **One log in the problem…**

  1. Move it around until you have just the log on one side and a single number on the other side.
  2. Convert to exponential form to solve.

- **Two logs (with the same base) in the problem…**

  1. Move it around until one log is on one side and one log is on the other side.
  2. Drop the logs and solve the resulting equation.

- **Three or more logs in the problem…**

  1. Use properties to combine logs until you have only one or two logs left in the problem, then follow the instructions above!

- **Solving for *x* in the exponent…$a^x = b$**

  1. Make sure you start with an equation in the form $a^x = b$ (get rid of any additional numbers just like you would when solving an equation).
  2. Take a common log of each side
  3. Move the *x* (exponent) down in front and solve to get *x* alone.

Practice

1. $\cancel{\log_2} 3x = \cancel{\log_2} 6$

$$\frac{\cancel{3}x}{\cancel{3}} = \frac{6}{3}$$

$$\boxed{x = 2}$$

2. $\cancel{\log_5}(-2a+9) = \cancel{\log_5}(7-4a)$

$$\cancel{-2a} + 9 = 7 - 4a$$
$$\cancel{+2a} \qquad\qquad +2a$$

$$9 = \cancel{7} - 2a$$
$$-7 \quad \cancel{-7}$$

$$\frac{2}{-2} = \frac{\cancel{-2}a}{\cancel{-2}}$$

$$\boxed{-1 = a}$$

3. $\log_9 x - \log_9 4 = \log_9 12$

$$\cancel{\log_9} \frac{x}{4} = \cancel{\log_9} 12$$

$$\cancel{4 \cdot} \frac{x}{\cancel{4}} = 12 \cdot 4$$

$$\boxed{x = 48}$$

4. $\log_3 x = -2$

convert

$$3^{-2} = x$$

$$\boxed{\frac{1}{9} = x}$$

5. $-\cancel{10}+\log_3(n+3)=-10$

$+\cancel{10} \qquad +10$

convert

$\log_3(n+3)=0$

$3^0 = n+3$

$1 = n+\cancel{3}$

$-3 \qquad \cancel{-3}$

$\boxed{-2=n}$

6. $-6\log_3(x-3)+\cancel{5}=-19$

$-\cancel{5} \qquad -5$

$\frac{\cancel{-6}\log_3(x-3)}{\cancel{-6}} = \frac{-24}{-6}$

$\log_3(x-3)=4$

convert

$3^4 = x-3$

$81 = x-3$

$+3 \qquad +3$

$\boxed{84=x}$

7. $3^x = 10$

$\log 3^x = \log 10$

$\frac{x\cancel{\log 3}}{\cancel{\log 3}} = \frac{\log 10}{\log 3}$

$\boxed{x = 2.096}$

8. $5^x + \cancel{3} = 59$

$-\cancel{3} \qquad -3$

$5^x = 56$

$\log 5^x = \log 56$

$\frac{x\cdot\cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 56}{\log 5}$

$\boxed{x = 2.5}$

9. $\frac{\cancel{2}(5)^x}{\cancel{2}} = \frac{30}{2}$

$5^x = 15$

$\log 5^x = \log 15$

$\frac{x\cdot\cancel{\log 5}}{\cancel{\log 5}} = \frac{\log 15}{\log 5}$

$\boxed{x = 1.683}$