Name: KEY!

Hour: _____

Chapter 7

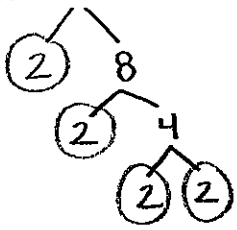
Powers & Exponents

Lesson 7-6: Simple Rational Exponents

Vocabulary

Fractional Exponents: splits the base into equal factors *not by division!

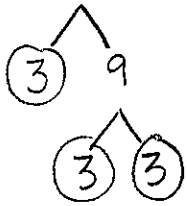
Example: $16^{1/4}$



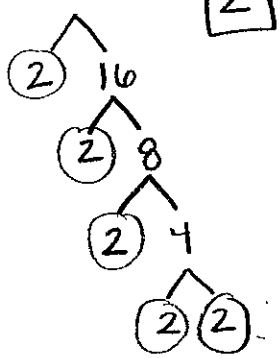
So, $16^{1/4} = \boxed{2}$

Practice

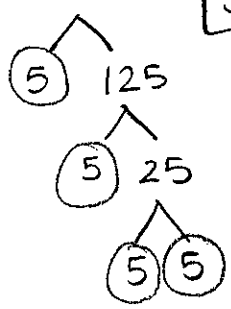
1. $27^{1/3} = \boxed{3}$



2. $32^{1/5} = \boxed{2}$



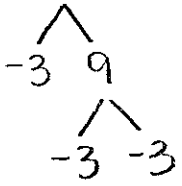
3. $625^{1/4} = \boxed{5}$



Tricky situations...

- If the fraction has an odd denominator, the base can be negative
- ***The answer will be negative.
- If the fraction has an even denominator, the base can NOT be negative

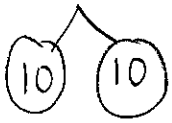
4. $-27^{1/3} = \boxed{-3}$



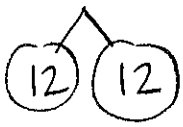
5. $-64^{1/2}$

not possible!
Two identical #'s can't be multiplied to get a neg.

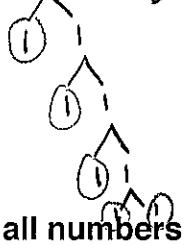
6. $100^{1/2} = \boxed{10}$



7. $144^{1/2} = \boxed{12}$



8. $1^{1/6} = 1$ — 1 to any power will = 1!



9. $\left(\frac{27}{125}\right)^{1/3} = \frac{27^{1/3}}{125^{1/3}} = \frac{3}{5}$

Not all numbers split evenly...

- If a number does NOT split evenly into the correct number of factors, use a calculator to get a decimal approximation.
- When you type these into a calculator, put the exponent, which is a fraction, in parentheses.
- If the base is negative, put the base into parentheses also!
- Always round to three decimal places!

10. $10^{1/3} =$ 2.154

11. $52^{1/7} =$ 1.759

12. $-12^{1/6} =$ ∅ (error)

Solving equations with Rational ("Fraction") Exponents...

- To get rid of a fractional exponent, raise to the power of the reciprocal!

13. $x^{1/8} = 7$

$(x^{1/8})^8 = 7^8$

$x = 5,764,801$

14. $\frac{-10x^{1/3}}{-10} = \frac{40}{-10}$

$x^{1/3} = -4$

$(x^{1/3})^3 = (-4)^3$

$x = -64$

Lesson 7-7 & 7-8: Complex Rational Exponents

Practice

ALL fractions CAN be split...

1. $25^{\frac{3}{2}}$ ($\frac{1}{2}\cdot 3$)

$(25^{1/2})^3$

$5^3 = \boxed{125}$

2. $64^{\frac{5}{6}}$ ($\frac{1}{6}\cdot 5$)

$(64^{1/6})^5$

$2^5 = \boxed{32}$

3. $-1024^{\frac{4}{5}}$ ($\frac{1}{5}\cdot 4$)

$-(1024^{\frac{1}{5}})^4$

$-(4)^4$

$\boxed{-256}$

4. $\left(\frac{64}{9}\right)^{\frac{5}{2}}$ ($\frac{1}{2}\cdot 5$)

$\frac{(64^{\frac{1}{2}})^5}{(9^{\frac{1}{2}})^5} = \frac{8^5}{3^5}$

$= \boxed{\frac{32,768}{243}}$

5. $\left(x^{\frac{2}{3}}y^{\frac{1}{2}}\right)^{\frac{2}{5}}$ * Power to a Power... multiply!

$\boxed{x^{\frac{4}{15}}y^{\frac{2}{10}}}$

6. $(16a^4)^{\frac{3}{4}}$

$16^{\frac{3}{4}}a^{\frac{12}{4}}$

$(16^{\frac{1}{4}})^3 a^3$

$2^3 a^3 = \boxed{8a^3}$

What do NEGATIVE EXPONENTS do?!? Reciprocals!

7. $27^{1/3} = \boxed{3}$ → $27^{-1/3}$ $\boxed{\frac{1}{3}}$

8. $64^{\frac{4}{3}}$ ($\frac{1}{3}\cdot 4$) $= (64^{\frac{1}{3}})^4$

$4^4 = \boxed{256}$ → $64^{-4/3}$ $\boxed{\frac{1}{256}}$

9. $\left(\frac{49}{25}\right)^{\frac{3}{2}}$ ($\frac{1}{2}\cdot 3$)

$\frac{(49^{\frac{1}{2}})^3}{(25^{\frac{1}{2}})^3} = \frac{7^3}{5^3} = \boxed{\frac{343}{25}}$ → $\left(\frac{49}{25}\right)^{-3/2}$ $\boxed{\frac{25}{343}}$

Lesson 7-4: Compound Interest

Vocabulary

Compound Interest: interest is being combined in over & over & over again.

Compound Interest Formula: $A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$

P = Principal (initial investment) A = Total Amount in Account n = # of times compounded per year

r = interest rate (decimal version) t = time (in years)

Practice

1. $3000 (P) is invested at 3.2% (.032 r) compounded quarterly (4 times per year n). Find the total amount in the account after 5 years (t).

$$A = 3000\left(1 + \frac{.032}{4}\right)^{4 \cdot 5}$$ (20)

$$A = \$3518.29$$

2. $41,248 (P) is invested at 4.6% (.046 (r)) compounded semi-annually (2 times n). Find the total amount in the account after 54 months. (4.5 years (t))

$$A = 41{,}248\left(1 + \frac{.046}{2}\right)^{2 \cdot 4.5}$$ (9)

$$A = \$50{,}615.51$$ (A)

3. An account currently has $7240 (A) in it. The account was opened 6 years (t) ago at 2.3% (.023 r) compounded monthly (12 times n). How much money was invested initially?

$$7240 = P\left(1 + \frac{.023}{12}\right)^{12 \cdot 6}$$ (calc.)

$$\frac{7240}{1.148} = \frac{P \cdot 1.148}{1.148}$$

$$\$6306.62 = P$$

Lesson 7-1: Power Functions

Vocabulary

Power Function: $y = x^n$ (power: n, exponent; base: x)

EVEN Power Functions: ex: $y = x^4$

- Graph is "U-shaped" - similar to parabola.
- Domain = all real #'s
- Range = positive real #'s
- Symmetric over the y-axis (with a vertical line).

ODD Power Functions: ex: $y = x^5$

- Graph is "squiggle" shaped
- Domain = all real #'s
- Range = all real #'s
- Rotation-symmetry around the origin.

Practice

1. Luisa is taking a quiz. Unfortunately, she didn't study or pay attention in class, so she plans to simply guess on each of the five questions. Suppose the probability of getting a question right is *p*. What is the probability that she will score 100%?

$$\frac{P}{\#1} \cdot \frac{P}{\#2} \cdot \frac{P}{\#3} \cdot \frac{P}{\#4} \cdot \frac{P}{\#5} = \boxed{P^5}$$

If this were a true/false quiz ($\frac{1}{2}$), what probability would she have of getting 100%?

$$\left(\frac{1}{2}\right)^5 = \frac{1^5}{2^5} = \boxed{\frac{1}{32}}$$

If this were a multiple choice quiz with 5 choices ($\frac{1}{5}$) for each question, what probability would she have of getting 100%?

$$\left(\frac{1}{5}\right)^5 = \frac{1^5}{5^5} = \boxed{\frac{1}{3125}}$$

2. The point (-2, 64) is on the graph of a power function.

Is this function even or odd? Why?

Even - the negative # turned positive

Does the graph have a minimum or maximum? If so, what is it?

Minimum - at zero

Write an equation for the function. must be even!

Guess & check: $(-2)^2 = 64$

$(-2)^6 = 64$ So, $\boxed{y = x^6}$

3. The point (-4, -1024) is on the graph of a power function.

Is this function even or odd? Why?

Odd - the negative # stayed negative

Does the graph have a minimum or maximum? If so, what is it?

NO - extends forever in both directions.

Write an equation for the function.

Guess & check: $(-4)^? = -1024$ → must be odd

$(-4)^5 = -1024$

So, $\boxed{y = x^5}$