Name: KEY!

Hour: _____

Chapter 6

Part 1

QUADRATICS

Lesson 6-1: Multiplying Binomials

Vocabulary

Binomial: a polynomial w/ two terms

ex: $2x+1$ or $5+3x$ or $2y-3$, etc...

$(2x+1)(x-3)$

F - First

O - Outside

I - Inside

L - Last

F: $2x \cdot x = 2x^2$

O: $2x \cdot -3 = -6x$

I: $1 \cdot x = x$

L: $1 \cdot -3 = -3$

$2x^2 - 6x + x - 3$

combine

Simplify --> $\boxed{2x^2 - 5x - 3}$

To 'square' means... to multiply something by itself

Practice

1. Expand and Simplify $(x-3)^2$

$(x-3)(x-3)$

FOIL: $x^2 - 3x - 3x + 9$

$\boxed{x^2 - 6x + 9}$

2. Expand and Simplify $(2m+4)^2$

$(2m+4)(2m+4)$

FOIL: $4m^2 + 8m + 8m + 16$

$\boxed{4m^2 + 16m + 16}$

3. Expand and Simplify $(b+2)^2 - (b-2)^2$

$(b+2)(b+2) - (b-2)(b-2)$

$(b^2 + 2b + 2b + 4) - (b^2 - 2b - 2b + 4)$

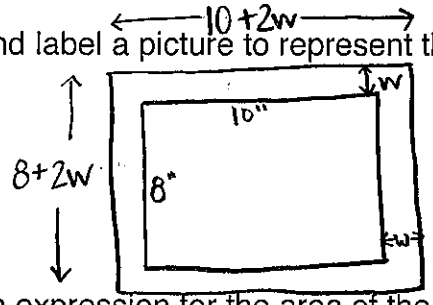
$b^2 + 4b + 4 - (b^2 - 4b + 4)$

$\longrightarrow \boxed{8b}$

RECTANGLE PROBLEMS

4. Ella has an 8" by 10" photograph that she wishes to frame with a matte that is *w* inches wide.

a. Draw and label a picture to represent this situation.



b. Write an expression for the area of the photo and frame.

↳ multiply both sides

$(8+2w)(10+2w)$

FOIL: $80 + 16w + 20w + 4w^2 = \boxed{80 + 36w + 4w^2}$

c. If she chooses to use a matte that is 4" wide, what will be the dimensions of the frame?

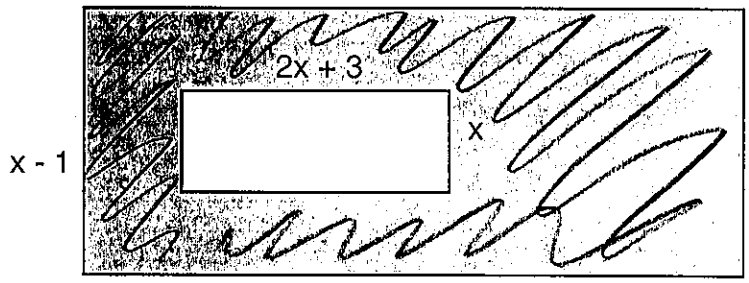
Plug 4 in for w

$10+2\cdot4 = 18''$

$8+2\cdot4 = 16''$

$\boxed{18'' \text{ by } 16''}$

5. Consider the diagram given.



a. Write a simplified expression for the area of the inner rectangle.

$(2x+3)(x) = \boxed{2x^2+3x}$

b. Write a simplified expression for the area of the outer rectangle.

$(x-1)(x+12)$

FOIL:

$x^2 + 12x - 1x - 12 = \boxed{x^2+11x-12}$

c. Write a simplified expression for the shaded region.

Big Rectangle − Small Rectangle

$(x^2+11x-12) - (2x^2+3x)$

$x^2 + 11x - 12 - 2x^2 - 3x = \boxed{-1x^2+8x-12}$

Lesson 6-2: Solving w/ Square Roots

*Every number has two square roots, so every equation has TWO SOLUTIONS!

Practice

1. $\sqrt{x^2} = \sqrt{16}$

$\boxed{x = 4 \text{ or } x = -4}$

2. $x^2 - \cancel{5} = 139$

$\cancel{+5} \quad +5$

$\sqrt{x^2} = \sqrt{144}$

$\boxed{x = 12 \text{ or } x = -12}$

3. $4x^2 + \cancel{1} = 17$

$\cancel{-1} \quad -1$

$\frac{4x^2}{4} = \frac{16}{4}$

$\sqrt{x^2} = \sqrt{4}$

$\boxed{x = 2 \text{ or } x = -2}$

4. $\sqrt{(x-3)^2} = \sqrt{64}$

$x - \cancel{3} = 8$ or $x - \cancel{3} = -8$

$\cancel{+3} \quad +3 \qquad \cancel{+3} \quad +3$

$\boxed{x = 11 \text{ or } x = -5}$

5. $\frac{\cancel{6}(x+1)^2}{\cancel{6}} = \frac{6}{6}$

$\sqrt{(x+1)^2} = \sqrt{1}$

$x + \cancel{1} = 1$ or $x + 1 = -1$

$\cancel{-1} \quad -1 \qquad -1 \quad -1$

$\boxed{x = 0 \text{ or } x = -2}$

Lesson 6-3: The Graph Translation Theorem

Graph each on your calculator one at a time and compare...

	$y = x^2$	$y = (x - 2)^2 + 4$	$y = (x + 6)^2 + 3$
Vertex:	$(0, 0)$	$(2, 4)$	$(-6, 3)$

Vertex Form of a Parabola - The vertex will be (h, k)

Vertex Form: $* \boxed{y - k = a(x - h)^2} *$

(or you could move "k" over to the other side...)

$$y - \cancel{k} = (x - h)^2$$
$$+\cancel{k} \qquad\qquad +k$$
$$\boxed{y = a(x - h)^2 + k}$$

Practice

1. Find the image of $y = \overset{a}{5}x^2$ under $T_{\overset{h}{-\frac{2}{3}}, \overset{k}{6}}$

$$y - 6 = 5\left(x + \frac{2}{3}\right)^2$$

2. Find the image of $y = \overset{a}{5}x^2$ under $T_{\overset{h}{3}, \overset{k}{-8}}$

$$y + 8 = 5(x - 3)^2$$

3. Find the vertex of the graph with the equation $y \overset{k}{\underset{-5}{+5}} = 5(x \overset{h}{\underset{7}{-7}})^2$.

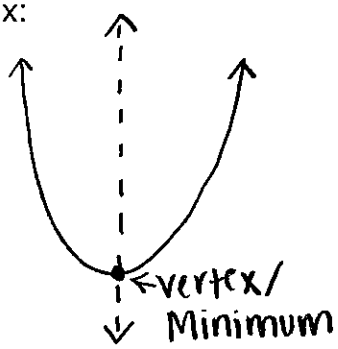
$\hookrightarrow (h, k)$

$\Downarrow$

$(7, -5)$

Properties of Parabolas

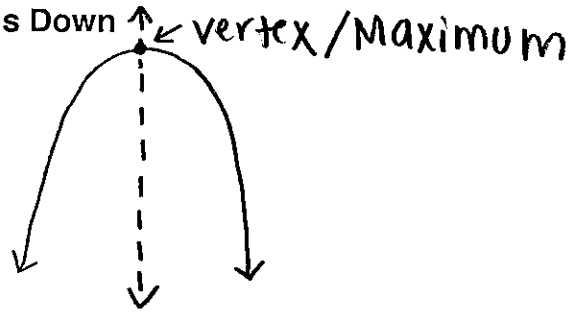
Opens Up
Ex:



Axis of Symmetry
Divides the parabola in half; goes through the vertex

Equation: $x = h$

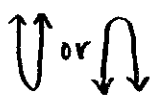
Opens Down
Ex:



Width of Parabola (ignore the positive/negative sign!!!)

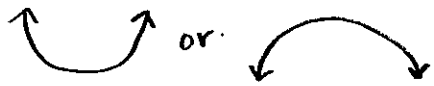
SKINNY if...

$|a| > 1$ the parabola will be SKINNY



WIDE if...

$|a| < 1$ the parabola will be WIDE!



Practice

4. The graph of $y = \overset{a}{4}x^2$ is translated $\underbrace{\text{2 units to the left}}_{h=-2}$ and $\underbrace{\text{5 units up}}_{k=5}$.

   a. Write an equation for its image.

   $$y - 5 = 4(x+2)^2$$

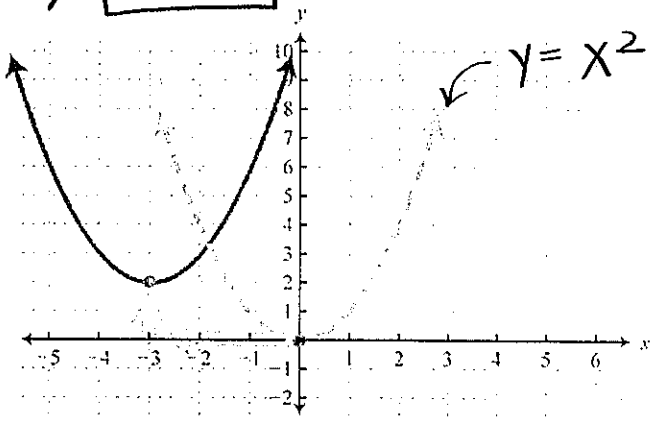
   b. The point (-1, 4) is a point on the preimage. Find the corresponding point on the image.

   $$\begin{array}{l} (-1, 4) \\ +(-2, 5) \end{array} = \boxed{(-3, 9)}$$

   c. Write the equation for the axis of symmetry.

   $x = h$ so, $\boxed{x = -2}$

5. Use the figure to the right.



   a. What translation maps the preimage to the image?

   left 3, up 2
   $(-3, 2)$ $\boxed{T_{-3,2}}$

   b. Write an equation for the solid parabola.

   $$y - 2 = (x+3)^2$$

Lesson 6-4: Converting Quadratics to Standard Form

Vocabulary

Standard Form for a quadratic looks like:

$$\boxed{y = ax^2 + bx + c}$$

BUT...you can't look at an equation and determine a, b, and c. SO...

To turn an equation into Standard Form from Vertex Form

1. Get "y" alone
2. FOIL & simplify

Practice

Convert each equation into standard form.

1. $y = 2(x-4)^2 + 1$

$y = 2(x-4)(x-4) + 1$

$y = 2(x^2 - 4x - 4x + 16) + 1$

$y = 2(x^2 - 8x + 16) + 1$

$y = 2x^2 - 16x + 32 + 1$

$\boxed{y = 2x^2 - 16x + 33}$

2. $y + 2 = \frac{1}{2}(x-6)^2 - 2$

$-2 \quad -2$

$y = \frac{1}{2}(x-6)(x-6) - 2$

$y = \frac{1}{2}(x^2 - 6x - 6x + 36) - 2$

$y = \frac{1}{2}(x^2 - 12x + 36) - 2$

$y = \frac{1}{2}x^2 - 6x + 18 - 2$

$\boxed{y = \frac{1}{2}x^2 - 6x + 16}$

3. $y = (x-3)^2 + 2$

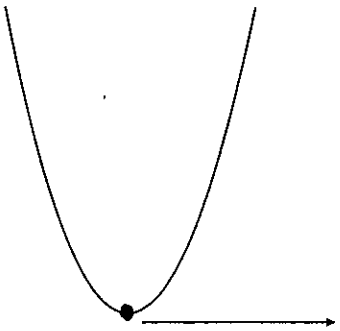
$y = (x-3)(x-3) + 2$

$y = (x^2 - 3x - 3x + 9) + 2$

$y = x^2 - 6x + 9 + 2$

$\boxed{y = x^2 - 6x + 11}$

Lesson 6-5: Converting Quadratics to Vertex Form



Vertex (h, k)

Standard Form: $y = ax^2 + bx + c$

$y = 3x^2 + 12x - 5$

a = 3
b = 12
c = -5

Finding the Vertex (h, k):

1. Find "h"

$$h = \frac{-b}{2a} \longrightarrow \frac{-12}{2 \cdot 3} = \frac{-12}{6} = -2$$

h = -2

2. Find "k" by plugging what you got for "h" into the original equation in place of "x".

$y = 3x^2 + 12x - 5$

k = $\underbrace{3(-2)^2 + 12(-2) - 5}_{\text{calculator}} = -17$

k = -17

So...the vertex is: $(-2, -17)$.
h k

Writing an equation in Vertex Form:

3. Plug "h" and "k" into the formula below for vertex form. Use "a" from the original equation that you had in standard form.

$$y - k = a(x - h)^2$$

$$\boxed{y + 17 = 3(x + 2)^2}$$

h = -2
k = -17
a = 3

Practice

Find the vertex. (h,k)

1. $y = x^2 - 4x + 1$

$a = 1$
$b = -4$
$c = 1$

① $h = \frac{-b}{2a} \rightarrow \frac{--4}{2 \cdot 1} = \frac{4}{2} = 2$, so $h = 2$

② $y = x^2 - 4x + 1$

$k = (2)^2 - 4(2) + 1$

$k = -3$

③ vertex

$\boxed{(2,-3)}$

Write in vertex form. $y - k = a(x-h)^2$

2. $y = -2x^2 - 8x + 1$

$a = -2$
$b = -8$
$c = 1$

① $h = \frac{-b}{2a} \rightarrow \frac{--8}{2 \cdot -2} = \frac{8}{-4} = -2$, so $h = -2$

② $y = -2x^2 - 8x + 1$

$k = -2(-2)^2 - 8(-2) + 1$

$k = 9$

③ vertex Form

$y - k = a(x-h)^2$

$\boxed{y - 9 = -2(x+2)^2}$

3. $y = 4x^2 - x - 3$

$a = 4$
$b = -1$
$c = -3$

① $h = \frac{-b}{2a} \rightarrow \frac{--1}{2 \cdot 4} = \frac{1}{8} = .125$, so $h = .125$ or $1/8$

② $y = 4x^2 - x - 3$

$k = 4(.125)^2 - (.125) - 3$

$k = -3.0625$ or $-49/16$

③ vertex Form

$y - k = a(x-h)^2$

$\boxed{y + 3.0625 = 4(x - .125)^2}$

Lesson 6-7: The Quadratic Formula

Quadratic Equations

$$ax^2 + bx + c$$

We can solve equations with just "x"

We can solve equations with just "x^2"

BUT...we can't solve equations with BOTH "x" and "x^2"...*yet.*

The Quadratic Formula

*NOTE: equations must be in standard form and also set equal to zero BEFORE you can use the quadratic formula!

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

ALL quadratic equations will have **2** solutions!

Practice

1. $10x^2 - 13x = 3 \rightarrow 10x^2 - 13x - 3 = 0$

$-3 \quad -3$

$a = 10$

$b = -13$

$c = -3$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(10)(-3)}}{2(10)}$$

$$x = \frac{13 \pm \sqrt{289}}{20}$$

$$x = \frac{13 \pm 17}{20}$$

$$\frac{13+17}{20} \text{ or } \frac{13-17}{20}$$

$$\frac{30}{20} = 1.5 \text{ or } \frac{-4}{20} = -0.2$$

2. $(3x - 6)(-2x + 1) = 0$

$-6x^2 + 3x + 12x - 6 = 0$

$-6x^2 + 15x - 6 = 0$

$a = -6$

$b = 15$

$c = -6$

$$x = \frac{(-15) \pm \sqrt{(15)^2 - 4(-6)(-6)}}{2 \cdot (-6)}$$

$$x = \frac{-15 \pm \sqrt{81}}{-12}$$

$$x = \frac{-15 \pm 9}{-12}$$

$$\frac{-15+9}{-12} \text{ or } \frac{-15-9}{-12}$$

$$= .5 \text{ or } 2$$

3. $2x^2 + 5 = 3x$

$-3x \quad -3x$

$2x^2 - 3x + 5 = 0$

$a = 2$

$b = -3$

$c = 5$

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{3 \pm \sqrt{-49}}{4}$$

$x = 3 + \text{error}$

$\boxed{\text{none}}$

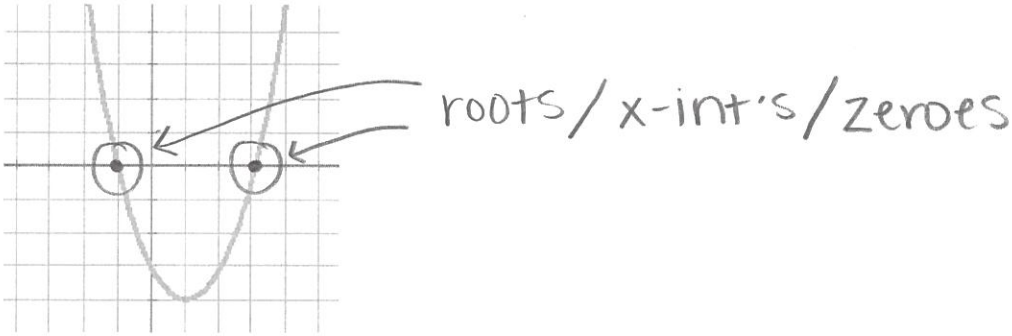
Lesson 6-10: The Nature of Roots

Vocabulary

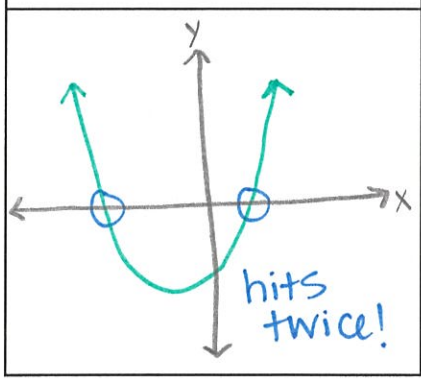
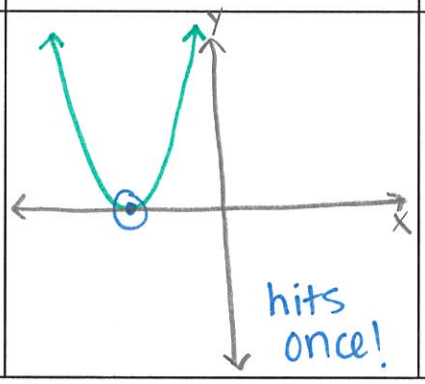
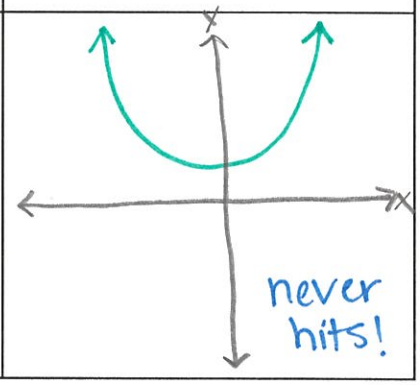
Roots/x-intercepts/zeroes: the point(s) where a graph crosses the x-axis

To find a root... plug 0 in for "y" & solve!

Example:



Types of Roots

Two Real Roots	One Real Root	No Real Roots
		

Discriminant: tells us how many real roots a quadratic has

Just look at the Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Discriminant is... $b^2 - 4ac$

If D > 0... two real roots

If D = 0... one real root

If D < 0... no real roots

↳ (two imaginary roots!)

Practice

1. Use the formula $y = 6x^2 - 3x - 4$.

   a. How many roots does the quadratic have? Show your work...

   ↳ use discriminant: $b^2 - 4ac$

   $a = 6$
   $b = -3$
   $c = -4$

   $(-3)^2 - 4(6)(-4) = 105$

   $\boxed{\text{So, there are two real roots}}$

   b. Find the root(s) (if possible).

   Quadratic Formula:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(6)(-4)}}{2(6)}$$

$$x = \frac{3 \pm \sqrt{105}}{12}$$

$$x = \frac{3 \pm 10.25}{12}$$

$$\frac{3 + 10.25}{12} \quad \text{or} \quad \frac{3 - 10.25}{12}$$

$$\frac{13.25}{12} \qquad \frac{-7.25}{12}$$

$$\boxed{\approx 1.1 \quad \text{or} \quad -0.6}$$