Name: KEY!

Hour: ____

Chapter 5

Systems

I want to listen to you but I'm thinking about snacks.

Lesson 5-1 Part 1: Solving & Graphing Inequalities

Solving Inequalities

★ Inequalities can be solved just like equations.

★ If you multiply or divide by a negative number, you MUST reverse the direction of the inequality symbol.

Graphing Inequalities

★ On a number line, use a closed circle ● if the inequality includes "equal to" (ex: ≤ or ≥). Use an open circle O if it does not (ex: > or <).

★ Shade in the direction of the inequality symbol as long as the variable is on the left side at the end!

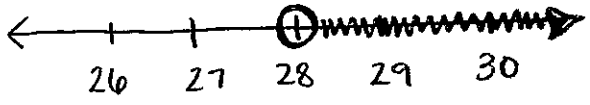
Practice

1. Solve $2m + 57 > 113$, and graph the solution set on a number line.

$-57 \quad -57$

$\frac{2m}{2} > \frac{56}{2}$

$m > 28$



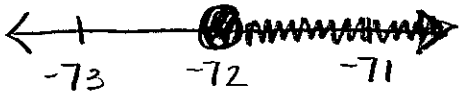
2. Solve $-3(x + 5) \leq 201$, and check your answer!

$-3x - 15 \leq 201$

$+15 \quad +15$

$\frac{-3x}{-3} \leq \frac{216}{-3}$) flip sign!

$x \geq -72$



3. A ticket agency has 275 tickets to a playoff game. Each caller receives 2 tickets. When there are 50 tickets or less remaining, the agency tries to obtain more tickets. How many callers can be served before more tickets are needed?

C = # of callers

$275 - 2 \cdot C \geq 50$

$-275 \qquad -275$

$\frac{-2C}{-2} \geq \frac{-225}{-2}$ ↘ flip sign!

$C \leq 112.5$

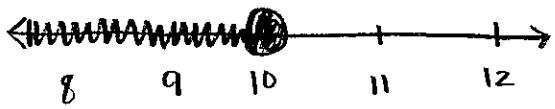
So, they can have 112 callers before needing to get more tickets.

4. Solve $3x - 4 \leq 26$, and graph the solution set.

$+4 \quad +4$

$\frac{3x}{3} \leq \frac{30}{3}$

$x \leq 10$



Lesson 5-1 Part 2: Compound Sentences

Vocabulary

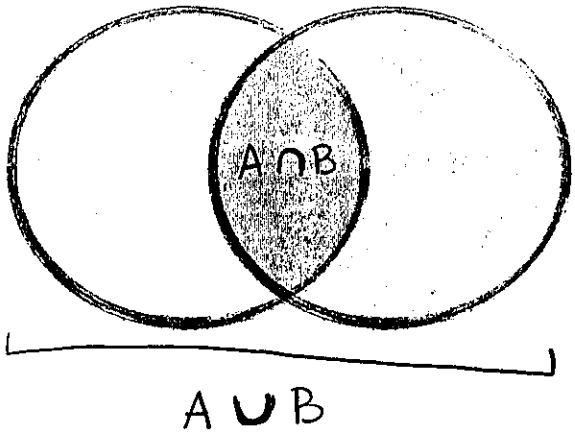
∩ → Intersection: the set of items SHARED by two sets at the same time

Example: The students who have Mrs. Merritt (AND) Mrs. Mossburger

∪ → Union: the set of items that belong to either set OR both (ALL items)

Example: The students who have Mrs. Merritt OR Mrs. Mossburger

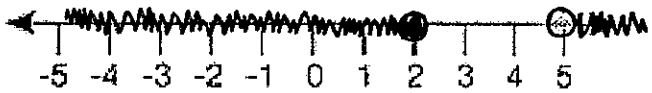
In a picture...



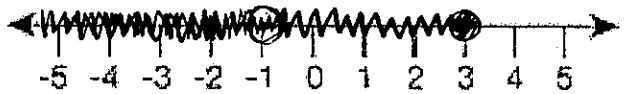
Practice

Sketch each on a number line.

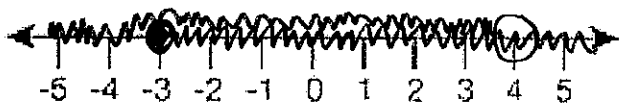
1. $x > 5$ (or) $x \leq 2$ ← ALL ∪



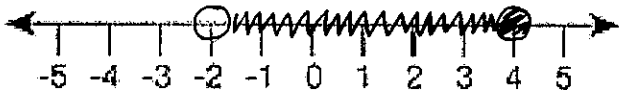
2. $x \leq 3$ (or) $x < -1$ ← ALL ∪



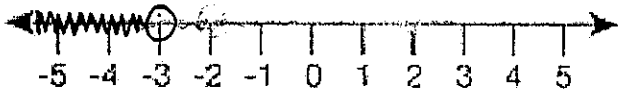
3. $x \geq -3$ (or) $x < 4$ ← ALL ∪



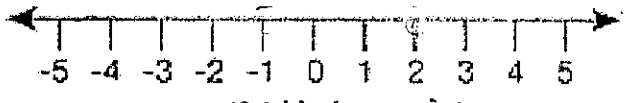
4. $x > -2$ and $x \leq 4$ ← shared



5. $x \leq 2$ and $x < -3$ ← shared



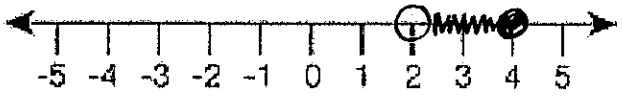
6. $x \geq 2$ and $x < -1$ ← shared



nothing is shared

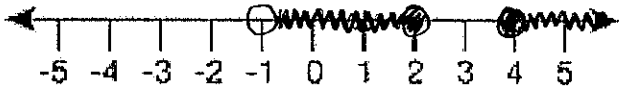
Set Notation

7. $\{x: 2 < x \leq 4\}$



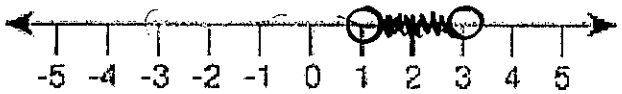
8. $\{m: -1 < m \leq 2\} \cup \{m: m \geq 4\}$

ALL



9. $\{p: -3 \leq p < 3\} \cap \{m: m > 1\}$

Shared

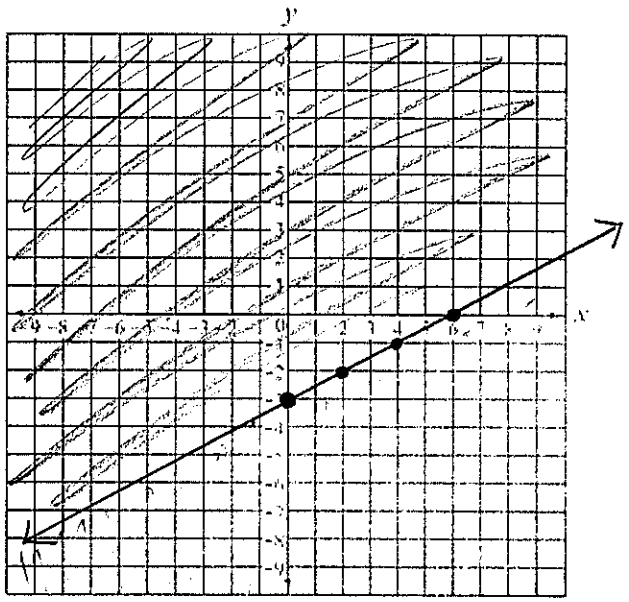


$y = mx + b$
↑ slope, start

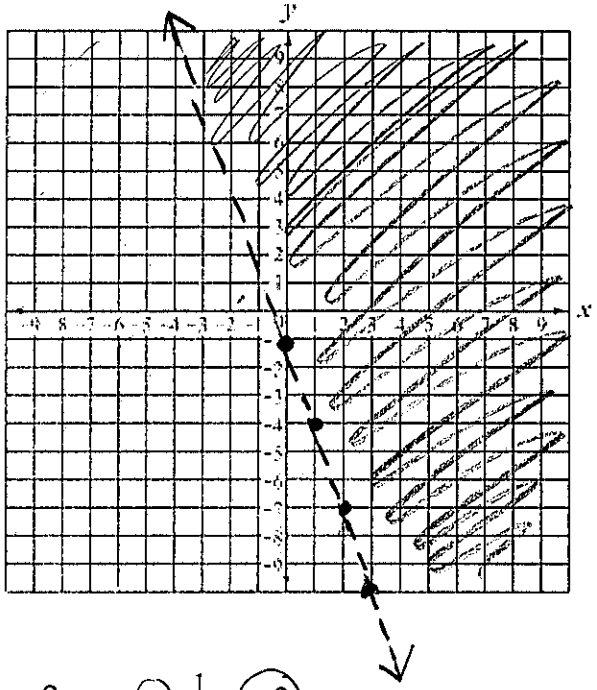
(≤) ← shade above or below
— ← dotted / solid line

Lesson 5-7: Graphing Linear Inequalities

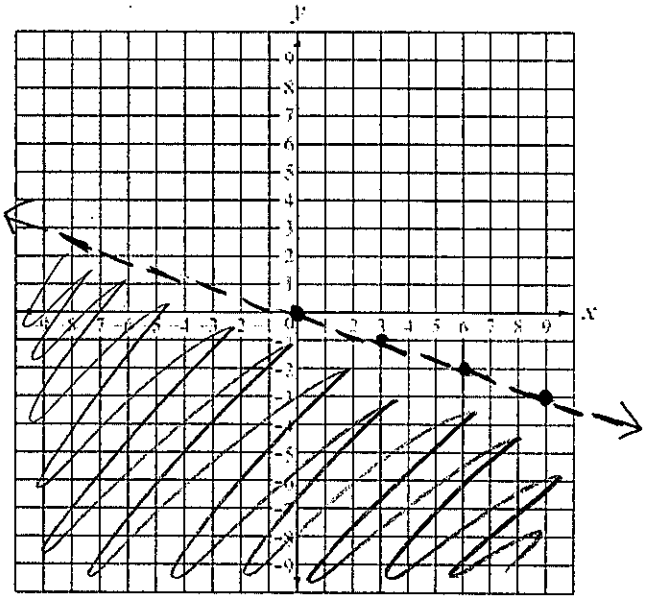
1. $y \leq \frac{1}{2}x - 3$ → below & solid; start; rise 1 / run 2



2. $y > -3x - 1$ → above & dotted; start; slope: rise −3 / run 1

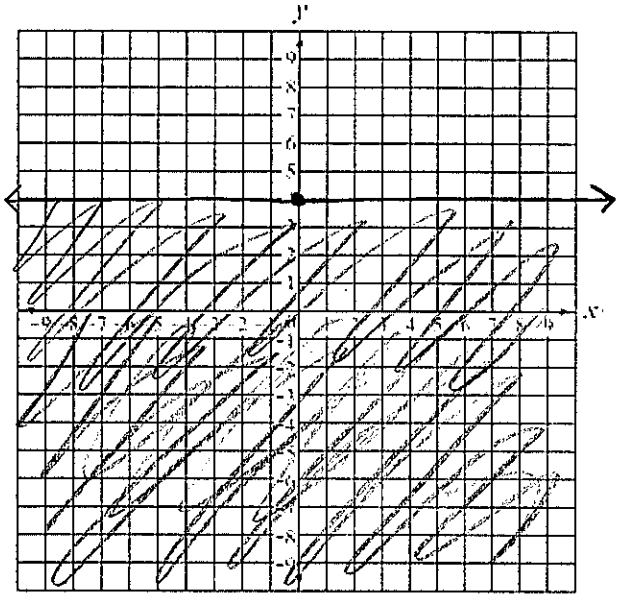


3. $y < -\frac{1}{3}x + 0$ → start; slope: rise −1 / run 3; below & dotted



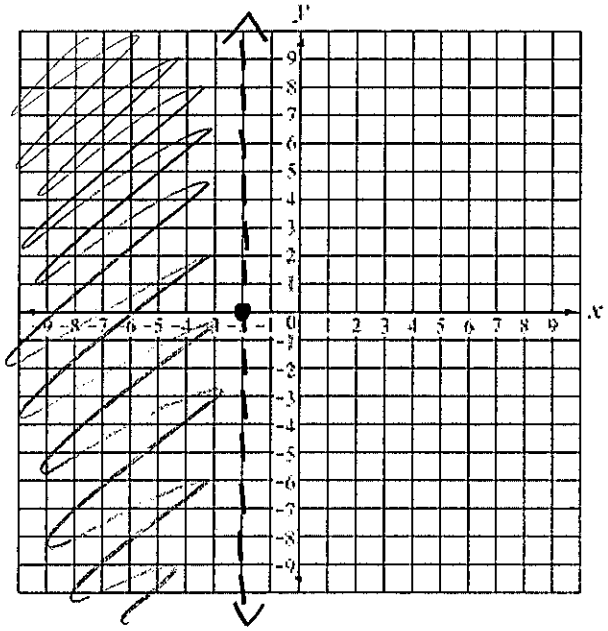
4. $y \leq 4$ ↱ below & solid

↥ WEIRDO! Horizontal Line!



5. $x < -2$ ↱ below & dotted

↥ WEIRDO! Vertical



6. $2x - 3y \geq 12$

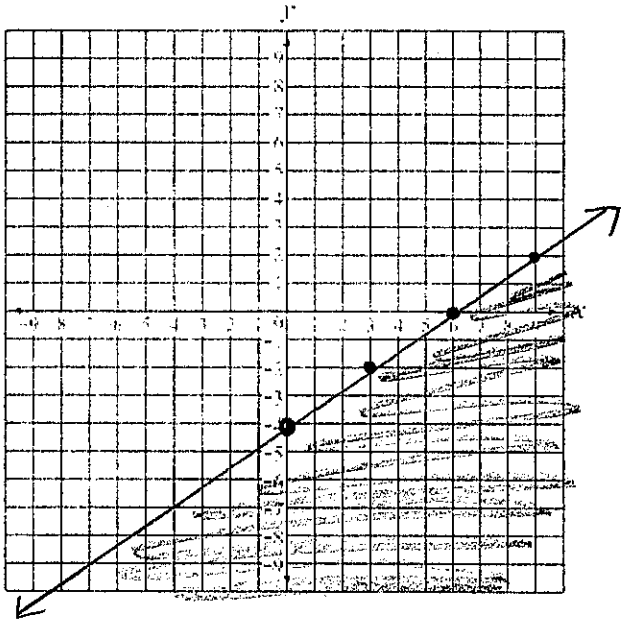
get "y" alone first!

$-2x \quad -2x$

$\frac{-3y}{-3} \geq \frac{12}{-3} \frac{-2x}{-3}$

$y \geq -4 + \frac{2}{3}x$

↥ above & solid ↥ start ↥ slope $\frac{\text{rise } 2}{\text{run } 3}$



Lesson 5-7 Continued...: Writing Linear Inequalities

Ask yourself...

1) What is the y-intercept? "b"

2) What is the slope? (rise/run) "m"

$y = mx + b$

3) Is the shaded area 'greater than' or 'less than'?
   - 'greater than' ($>$ or $\geq$) → shade above
   - 'less than' ($<$ or $\leq$) → shade below

4) Is the line dotted or solid? (solid: include equal bar, dotted: do not include equal bar)
   - $\leq$ or $\geq$ → solid
   - $<$ or $>$ → dotted

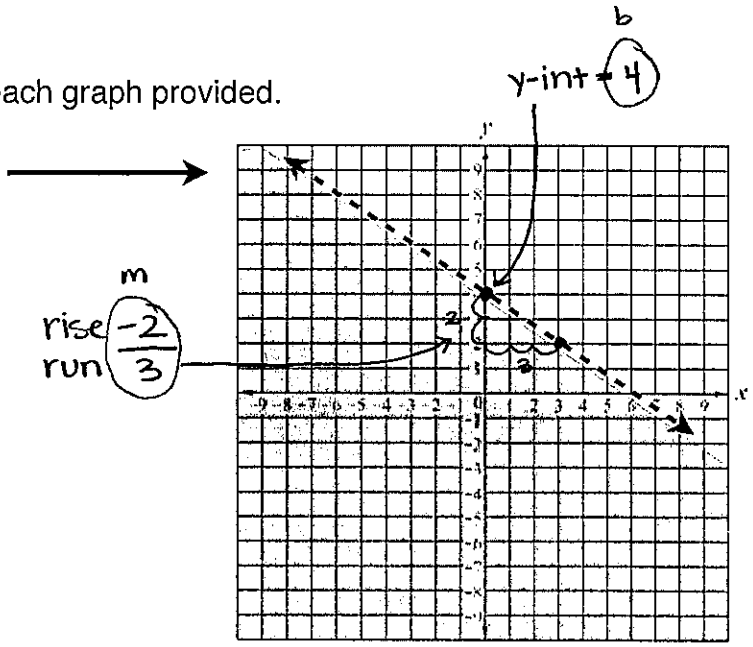
Practice

Write the equation for the line from each graph provided.

1. Equation:

$y < -\frac{2}{3}x + 4$

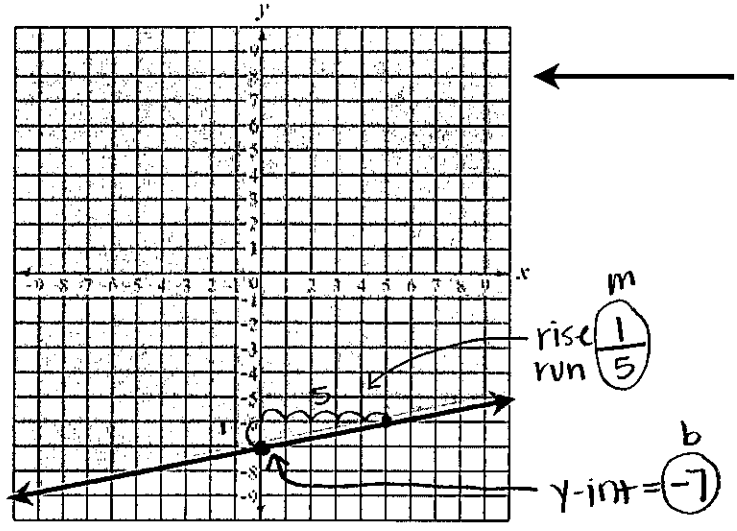
(< → below & dotted)



2. Equation:

$y \geq \frac{1}{5}x - 7$

(≥ → above & solid)



Lesson 5-2: Solving Systems by GRAPHING

Vocabulary

System of Linear Equations: a set of 2 or more linear equations w/ 2 or more variables.

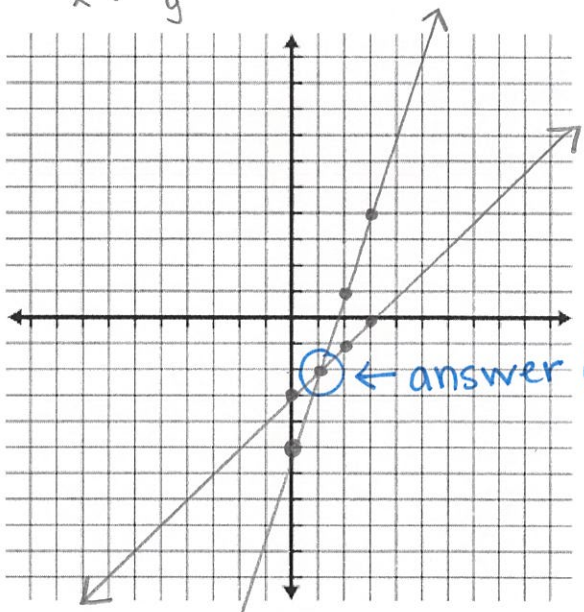
Solution to a System: an ordered pair that makes both equations true : (# , #) ← answer
(x, y)

Practice

1. slope $\frac{3}{1}$ ↓ start

$$\begin{cases} y = 3x - 5 \\ y = 1x - 3 \end{cases}$$

↑ slope $\frac{1}{1}$ start



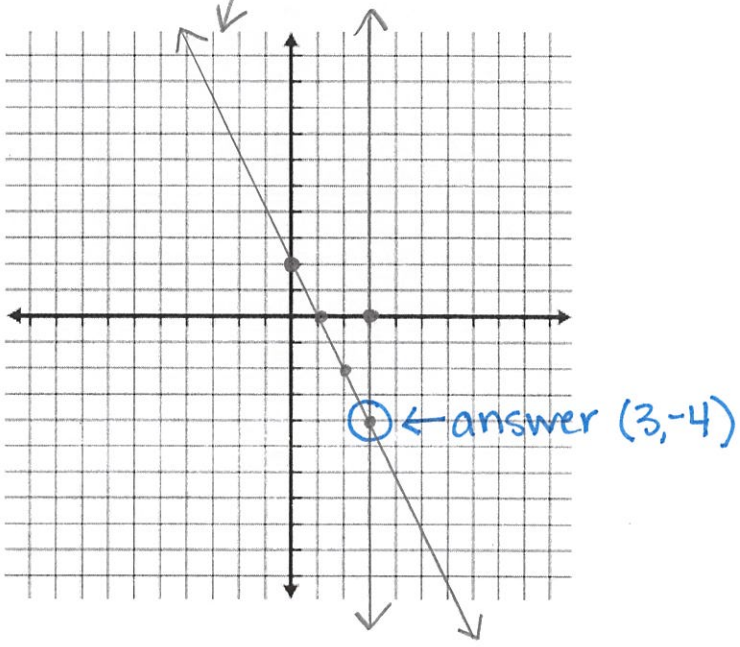
The two lines intersect at their answer,

(1, -2)

2. $\begin{cases} x = 3 \leftarrow \text{weird o, vertical} \\ y = -2x + 2 \end{cases}$

↑ start

Slope $\frac{-2}{1}$



3, -4

3. $\begin{cases} 6x = 5y \\ 4x - 2y = -8 \end{cases}$

*get y alone first!

$\frac{6x}{5} = \frac{5y}{5}$

$\frac{6}{5}x = y \rightarrow y = \frac{6}{5}x + 0$ (start)

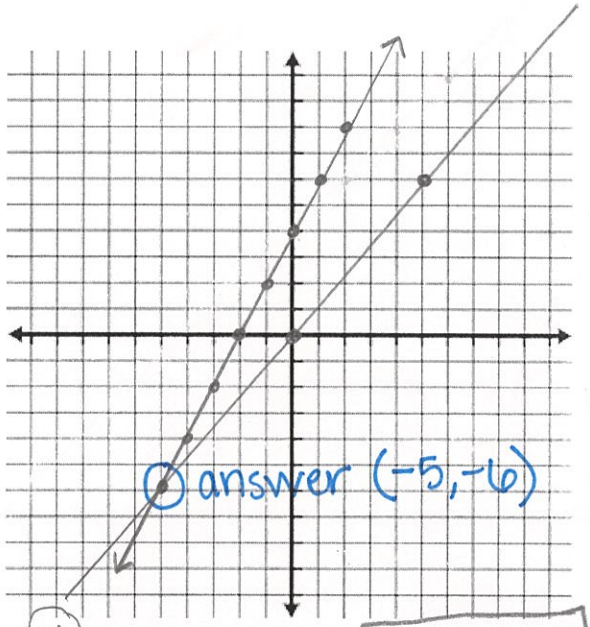
↑ slope

$4x - 2y = -8$

$-4x \quad -4x$

$\frac{-2y}{-2} = \frac{-4x}{-2} - \frac{8}{-2} \rightarrow y = 2x + 4$ (start)

↑ slope 2/1



(-5,-6)

Graphing Systems on your CALCULATOR

*Remember, to put an equation in your calculator it MUST be solved for y.

STEPS:

1) Press **Y =** . Type the equations in under **Y1** and **Y2**.
2) Press **GRAPH**. Make sure you can see both lines and the intersection point on your screen.
   *If you can't, try ZOOM-STANDARD or ZOOM-FIT.
3) Press **CALC** (Located above the TRACE key)
4) Choose #5 - **INTERSECT**
5) Press **ENTER** to confirm <u>each</u> line. Press **ENTER** one more time when your calculator says <u>Guess?</u>
6) Your solution will appear at the bottom of the screen!

Practice

1. $\begin{cases} y = 3x + 2 \checkmark \leftarrow \text{put in calc} \\ 2x - y = -3 \end{cases}$

get y alone

$-2x \quad -2x$

$-y = -2x - 3$

$y = 2x + 3$

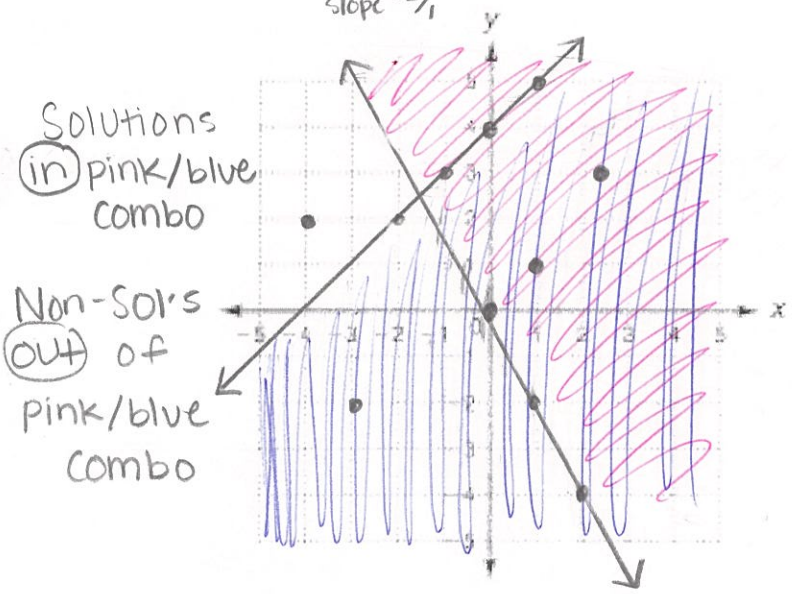
(1, 5)

Lesson 5-8: Graphing Systems of Linear Inequalities

Practice

Graph each system below. Then
(a) Identify two points that ARE solutions to the system, and
(b) identify two points that ARE NOT solutions to the system.

1. $\begin{cases} y \le x + 4 \\ y \ge -2x + 0 \end{cases}$ (slope $1/1$; start 4; slope $-2/1$; start 0)



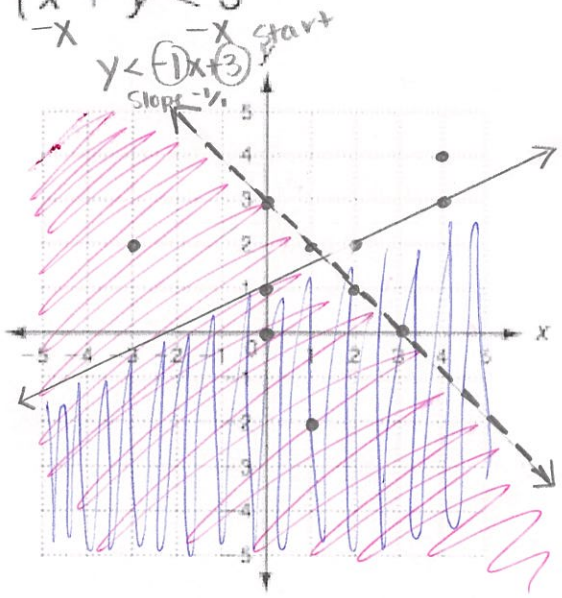
a. (1,1) & (2,3)

b. (-4,2) & (-3,-2)

2. $\begin{cases} y \le \frac{1}{2}x + 1 \\ x + y < 3 \end{cases}$ (slope $1/2$; start 1)

$-x \quad -x$

$y < -1x + 3$ (slope $-1/1$; start 3)



a. (0,0) & (1,-2)

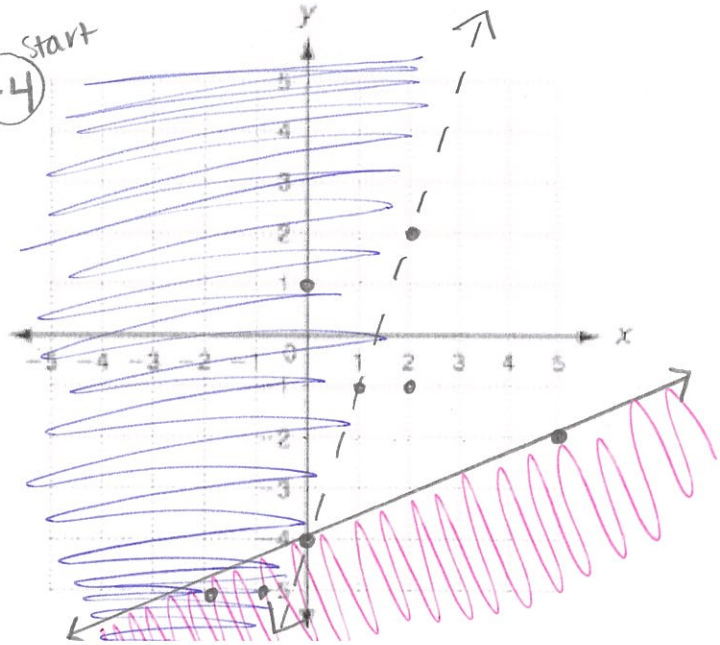
b. (-3,2) & (4,4)

3. $\begin{cases} y > 3x - 4 \\ 2x - 5y \ge 20 \end{cases}$ (slope $3/1$; start -4)

$-2x \quad -2x$

$\frac{-5y}{-5} \ge \frac{-2x}{-5} + \frac{20}{-5}$ *flip

$y \le \frac{2}{5}x - 4$ (slope $2/5$; start -4)



a. (-1,-5) & (-2,-5)

b. (2,-1) & (0,1)

4. The Biltrite Furniture Company makes wooden desks and chairs. Each item is worked on by a carpenter and a finisher. On average, carpenters spend four hours working on each chair and eight hours on each desk. There are enough carpenters for up to 8000 worker-hours per week. The finishers spend two hours on each chair and one hour on each desk. There are enough finishers for up to 1300 worker-hours per week. Given these constraints, find the feasible combinations of desks and chairs that can be completed in a week.

Step 1: Define Variables

C = chair

d = desk

Step 2: Organize the Information

4 hrs (per) chair (and) 8 hrs (per) desk ≤ 8000 hrs total

2 hrs (each) chair (&) 1 hr (each) desk ≤ 1300 hrs total

Step 3: Write Inequalities

$4c + 8d \leq 8000$ →

$2c + 1d \leq 1300$ →

get "d" alone

Step 4: Graph the Inequalities

$4c + 8d \leq 8000$

$-4c \quad -4c$

$\frac{8d}{8} \leq \frac{-4c}{8} + \frac{8000}{8}$

$d \leq (-\frac{1}{2})c + 1000$ (start)

slope

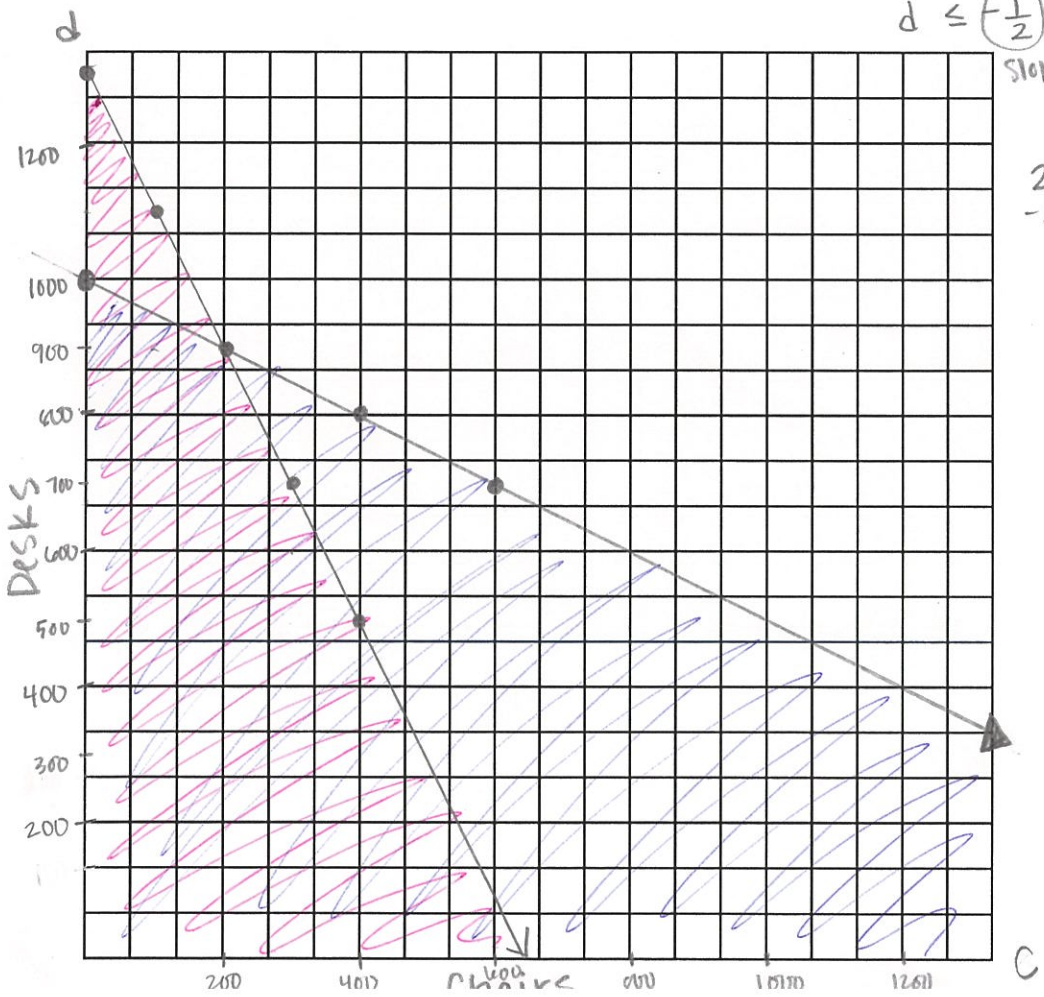
&

$2c + 1d \leq 1300$

$-2c \quad -2c$

$d \leq (-2)c + 1300$

slope $-2/1$ Sta



Lesson 5-3: Solving Systems by SUBSTITUTION

*To use this method, you must have ONE of the equations solved for ONE of the variables.

1) $\begin{cases} 3x + y = 25 \\ y = x - 3 \end{cases}$

$3x + x - 3 = 25$

$4x - 3 = 25$

$+3 \quad +3$

$\frac{4x}{4} = \frac{28}{4}$

$x = 7$

Plug $x = 7$ into one starting equation!

$y = x - 3$

$y = 7 - 3$

$y = 4$

Answer: (7, 4)

2) $\begin{cases} y = 3x \\ y = x + 4 \end{cases}$

$3x = x + 4$

$-x \quad -x$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

Plug into one of starting equations!

$y = 3x$

$y = 3 \cdot 2$

$y = 6$

Answer: (2, 6)

3) $\begin{cases} 2x + y = -1 \\ -3x + 3y = -21 \end{cases}$

Solve one equation for "y"

$2x + y = -1$

$-2x \quad -2x$

$y = -1 - 2x$

$-3x + 3(-1 - 2x) = -21$

$-3x - 3 - 6x = -21$

$-9x - 3 = -21$

$+3 \quad +3$

$\frac{-9x}{-9} = \frac{-18}{-9}$

$x = 2$

$2x + y = -1$

$2 \cdot 2 + y = -1$

$4 + y = -1$

$-4 \quad -4$

$y = -5$

Answer (2, -5)

4) $\begin{cases} y = 8 \\ 2x + 5y = 8 \end{cases}$

$2x + 5 \cdot 8 = 8$

$2x + 40 = 8$

$-40 \quad -40$

$\frac{2x}{2} = \frac{-32}{2}$

$x = -16$

$2x + 5y = 8$

$2 \cdot -16 + 5y = 8$

$-32 + 5y = 8$

$+32 \quad +32$

$5y = 40$

$\frac{5y}{5} = \frac{40}{5}$

$y = 8$

Answer (-16, 8)

Lesson 5-4: Solving Systems by ELIMINATION

*To use this method, you must have the variables lined up. One of the variables must have opposite coefficients, or the same coefficient.

1) $\begin{cases} x + 3y = 14 \\ 2x - 3y = -8 \end{cases}$ (+)

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$ Plug into one of the originals!

$x + 3y = 14$

$2 + 3y = 14$

$-2 \quad -2$

$3y = 12$

$\frac{3y}{3} = \frac{12}{3}$

$y = 4$

Answer: (2, 4)

2) $\begin{cases} 2x + 2y = 4 \\ 2x + 3y = 7 \end{cases}$ (−)

$y = -3$

$\frac{y}{-1} = \frac{-3}{-1}$

$y = 3$ Plug into one of the originals!

$2x + 2y = 4$

$2x + 2 \cdot 3 = 4$

$2x + 6 = 4$

$-6 \quad -6$

$2x = -2$

$\frac{2x}{2} = \frac{-2}{2}$

$x = -1$

Answer (-1, 3)

3) $\begin{cases} 2x - 6y = -10 \\ 9x + 6y = -78 \end{cases}$ (+) (3 × first equation)

$11x = -88$

$\frac{11x}{11} = \frac{-88}{11}$

$x = -8$ Plug in!

$2 \cdot x - 6y = -10$

$2 \cdot -8 - 6y = -10$

$-16 - 6y = -10$

$+16 \quad +16$

$-6y = 6$

$\frac{-6y}{-6} = \frac{6}{-6}$

$y = -1$

Answer: (-8, -1)

4) $\begin{cases} 3y - 2y = -6 \\ 3x + 7.5y = 22.5 \end{cases}$ (− after 1.5 × first equation)

$-9.5y = -28.5$

$\frac{-9.5y}{-9.5} = \frac{-28.5}{-9.5}$

$y = 3$ Plug in!

$2x + 5y = 15$

$2x + 5 \cdot 3 = 15$

$2x + 15 = 15$

$-15 \quad -15$

$2x = 0$

$\frac{2x}{2} = \frac{0}{2}$

$x = 0$

Answer: (0, 3)

5) $\begin{cases} y = 3x - 10 \\ 2x - y = 6 \\ 3x + y = -10 \end{cases}$ (+)

$-3x \quad -3x$

rearrange: $-3x + y = -10$

$-4x = -4$

$\frac{-4x}{-4} = \frac{-4}{-4}$

$x = 4$ Plug in!

$y = 3x - 10$

$y = 3 \cdot 4 - 10$

$y = 2$

Answer (4, 2)

6) $\begin{cases} 3x + 3y = -3 \\ 2x + 4y = -10 \end{cases}$ (− after 1.5 × second equation)

$3x + 3y = -3$

$3x + 6y = -15$

$-3y = 12$

$\frac{-3y}{-3} = \frac{12}{-3}$

$y = -4$ Plug in!

$3x + 3y = -3$

$3x + 3 \cdot -4 = -3$

$3x + -12 = -3$

$+12 \quad +12$

$3x = 9$

$\frac{3x}{3} = \frac{9}{3}$

$x = 3$

Answer (3, -4)

Lesson: Systems w/ Word Problems

Practice

1) Bill and Emily are both ordering fast food for their families while on a road trip. Bill orders 2 burgers and 3 orders of french fries. The total cost of his meal is $8.50. Emily orders 4 burgers and 2 orders of french fries. The total cost of her meal is $11. What is the cost of a burger and an order of fries?

Bill: $2\cdot(2B + 3F = 8.50) \rightarrow 4B + 6F = 17$

Emily: $4B + 2F = 11 \rightarrow 4B + 2F = 11$

$4F = 6$, $F = \$1.50$ (Plug in!)

$2B + 3F = 8.50$

$2B + 3\cdot 1.50 = 8.50$

$2B + 4.50 = 8.50$

$-4.50 \quad -4.50$

$2B = 4$, $B = \$2$

So, $F = \$1.50$ & $B = \$2$

2) There are 13 animals in a barn, some chickens and some pigs. There are 40 legs in all. How many of each animal are there?

$2\cdot(C + P = 13) \rightarrow 2C + 2P = 26$

$2C + 4P = 40 \rightarrow 2C + 4P = 40$

$-2P = -14$, $P = 7$ (Plug in!)

$C + P = 13$

$C + 7 = 13$, $C = 6$

So, 7 pigs & 6 chickens

3) Two schools are planning senior trips to New York City. This year the senior class at Grant High School rented (and filled) 1 van and 6 buses with 372 students. The senior class at Jefferson High School rented and filled 4 vans and 12 buses with 780 students. How many students can each van and each bus hold?

GHS: $4\cdot(1V + 6B = 372) \rightarrow 4V + 24B = 1488$

JHS: $4V + 12B = 780 \rightarrow 4V + 12B = 780$

$12B = 708$, $B = 59$ (Plug in!)

$1V + 6B = 372$

$1V + 6\cdot 59 = 372$

$V + 354 = 372$

$-354 \quad -354$

$V = 18$

So, bus holds 59, van holds 18

4) On a given day, 275 people attended the state fair. Adult tickets cost $8 and child tickets cost $5. On this particular day, the total ticket sales were $1975. How many of each type of ticket were sold?

$5\cdot(A + C = 275) \rightarrow 5A + 5C = 1375$

$8A + 5C = 1975 \rightarrow 8A + 5C = 1975$

$-3A = -600$, $A = 200$ (Plug in!)

$A + C = 275$

$200 + C = 275$

$-200 \quad -200$

$C = 75$

So, 200 adults, 75 children

5) The difference of two numbers is 7. The first number is 5 less than twice the second number. Find the two numbers.

$x - y = 7 \rightarrow x - y = 7$

$x = 2y - 5 \rightarrow x - 2y = -5$

$y = 12$ (Plug in!)

$x - y = 7$

$x - 12 = 7$

$+12 \quad +12$

$x = 19$

Lesson: Special Types of Systems

Vocabulary

Name of System Type	Description	Picture
Consistent	has a solution	
Inconsistent	no solution	
Independent	two lines are separate	
Dependent	two lines lay on top of one another	

How Can We Tell?!?! Just by looking at an equation...

Slope-Intercept Form

$y = mx + b$

No Solution

Same slope (m)

diff. y-intercept (b)

Infinitely Many Solutions

Same slope (m)

Same y-int (b)

Standard Form

$Ax + By = C$

No Solution

Same A & B #'s

diff. C #

Infinitely Many Solutions

Same A, B, & C #'s

Practice

1. SUB!

$$\begin{cases} y = 3x - 1 \\ y = \frac{1}{2}(6x - 8) \end{cases}$$

$3x - 1 = \frac{1}{2}(6x - 8)$

$3x - 1 = 3x - 4$

$-3x \quad -3x$

$-1 = -4 \ldots$ ☹ no solution

2. SUB!

$$\begin{cases} y = 2x - 4 \\ y = \frac{3}{4}\left(\frac{8}{3}x - \frac{16}{3}\right) \end{cases}$$

$2x - 4 = \frac{3}{4}\left(\frac{8}{3}x - \frac{16}{3}\right)$

$2x - 4 = 2x - 4$

$-2x \quad -2x$

$-4 = -4 \ldots$ ☺ infinite solutions

3. SUB!

$$\begin{cases} y - 1 = 5x \\ 2y = 10x + 2 \end{cases}$$

$y - 1 = 5x \rightarrow y = 5x + 1$

$+1 \quad +1$

$2y = 10x + 2$

$2(5x + 1) = 10x + 2$

$10x + 2 = 10x + 2$

$-10x \quad -10x$

$2 = 2 \ldots$ ☺ infinite solutions

4. SUB!

$$\begin{cases} x = 3 - 2y \\ 3x + 6y = 6 \end{cases}$$

$3(3 - 2y) + 6y = 6$

$9 - 6y + 6y = 6$

$9 = 6 \ldots$ ☹ no solutions

SOLVING SYSTEMS - Helpful Hints

Use SUBSTITUTION when...

* one variable is already alone!

Use ELIMINATION when...

* variables are lined up
* must have same ($-$) or opposite ($+$) coefficients

USE GRAPHING when...

* asked to! ☺