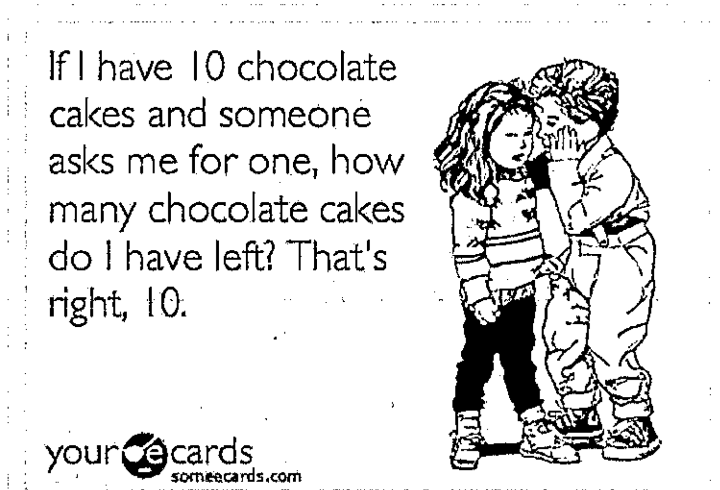
Name: KEY!

Hour: _____

Chapter 3
Linear Functions



Lesson 3-1: Linear Function Intro & Constant-Increase & Constant-Decrease Situations

Vocabulary

Linear Equation: a function that shows a relationship between two variables

Requirements:
- Variables must be in separate terms
- Variables cannot be located in the denominator
- Variables cannot have exponents

Linear Equations: Important Information

- A solution to a linear equation is a pair of numbers written as a coordinate (x,y)
- There are infinite solutions to a linear equation
- When solutions are graphed, the points form a straight line
- Linear equations can always be written in slope-intercept form

Slope-Intercept Form (remember this is just ONE way to write a linear equation, but there are other ways as well!)

$$y = \underbrace{m}_{\text{slope}}x + \underbrace{b}_{\text{y-intercept}}$$

- Slope = a measure of a line's steepness
- y-intercept = the point where a line cross the y-axis on a graph

Constant-Increase or Constant-Decrease Situations

- can be represented with a linear function
- two parts: initial value = y-intercept (b) & rate of change = slope (m)

Practice

1. Identify the equations that are considered linear equations.

$y = 3x + 1$	$2x + 3y = 9$	$y = 2x^2 + 7$
$4xy = 11$	$\frac{7}{x} = y$	$y = 12$

2. Find three solutions to the linear equation: $y = 3x - 2$.

$y = 3(0) + 2$
$y = 2$
① $(0, 2)$

$y = 3(1) + 2$
$y = 5$
② $(1, 5)$

$y = 3(2) + 5$
$y = 11$
③ $(2, 11)$

3. Identify the slope & y-intercept of each linear equation.

A. $y = \frac{2}{3}x - 5$

Slope = $2/3$

y-intercept = -5

B. $y = 2 - 4x$

Slope = -4 or $-4/1$

y-intercept = 2

4. An empty crate weighs 3 kilograms. It is filled with oranges that each weigh 0.2kg.

A. Is this is a constant-increase or constant-decrease situation?

B. Identify the initial value and rate of change.

Initial Value: 3 ← "b"
Rate Of Chg: 0.2 ← "m"

C. Write as a linear function.

$y = mx + b$

$\boxed{y = 0.2x + 3}$

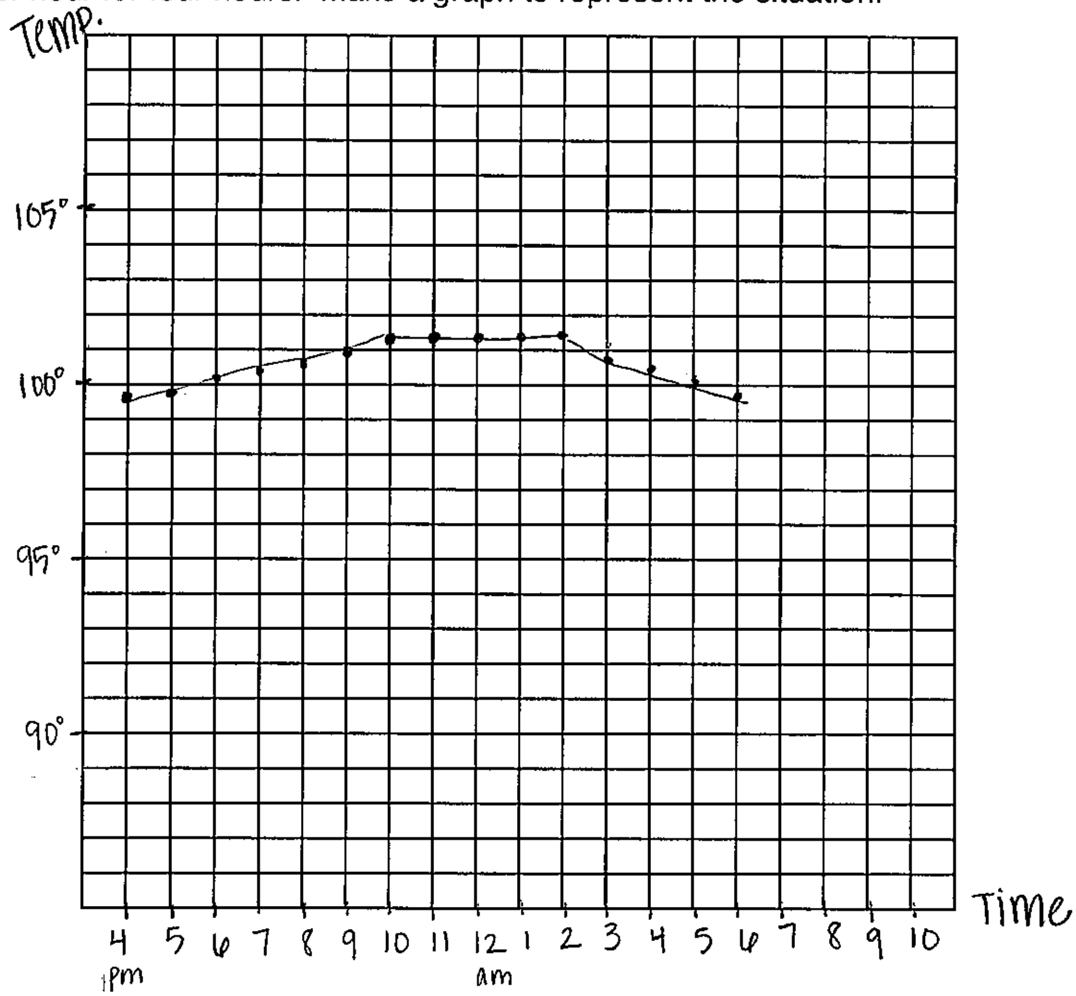
5. At the beginning of the month, Katie bough a 50 lb sack of wild bird feed. She puts 2/3 of a pound into the feeder each morning. Let y represent the amount of remaining feed after x days. Write a linear equation relating x and y.

Initial Value: 50 ← "b"
Rate of Chg: $-2/3$ ← "m"

$y = mx + b$

$\boxed{y = -\frac{2}{3}x + 50}$

6. Al's temperature at 4:00pm was 99.5°. It rose at a steady rate of 0.3° per hour for 6 hours. Then it stayed constant for four hours. Then, it fell steadily by 0.4° per hour for four hours. Make a graph to represent the situation.



4pm	5	6	7	8	9	10	11	12 am	1	2
99.5	99.8	100.1	100.4	100.7	101	101.3	101.3	101.3	101.3	101.3

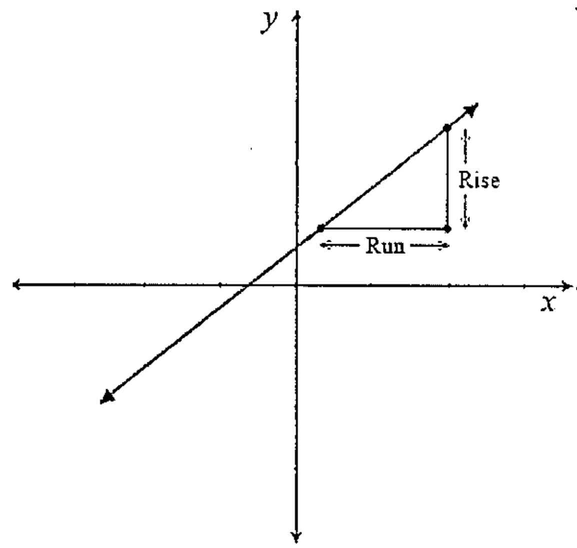
3	4	5	6
100.9	100.5	100.1	99.7

Lesson 3-2: The Graph of $y = mx + b$

Vocabulary

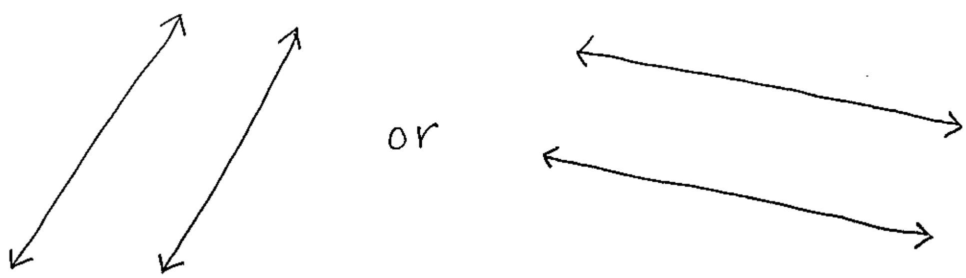
Slope: a ratio of a line's vertical change to its horizontal change

Slope = $\dfrac{\text{rise}}{\text{run}}$



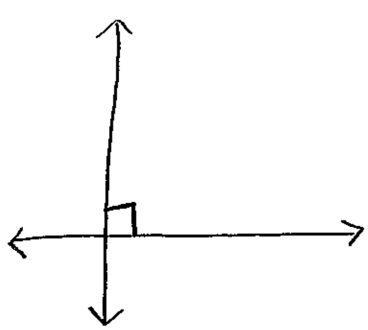
// Parallel Lines: two lines that never touch or are identical ; have SAME slope

Example:



$\perp$ Perpendicular Lines: two lines that form a 90° angle ; have opposite/reciprocal slopes

Example:



ex's: $\frac{1}{2}$ & -2

$3/4$ & $-4/3$

5 & $-1/5$

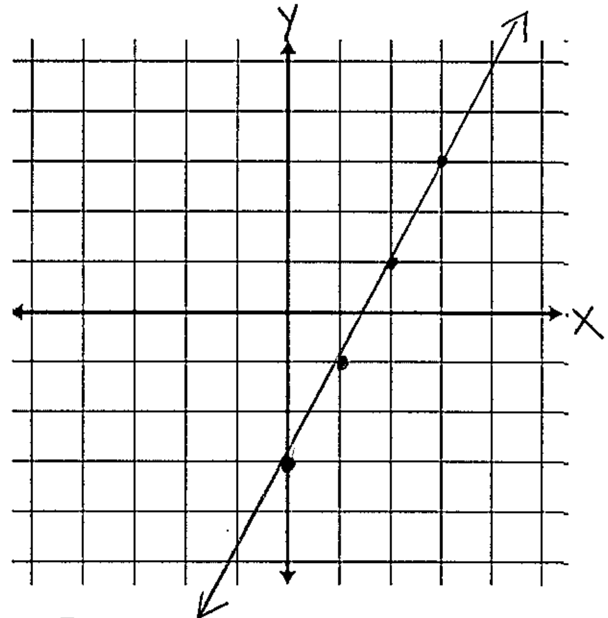
Practice

1. Graph the line $y = 2x - 3$. start

↳ slope $\frac{2}{1}$

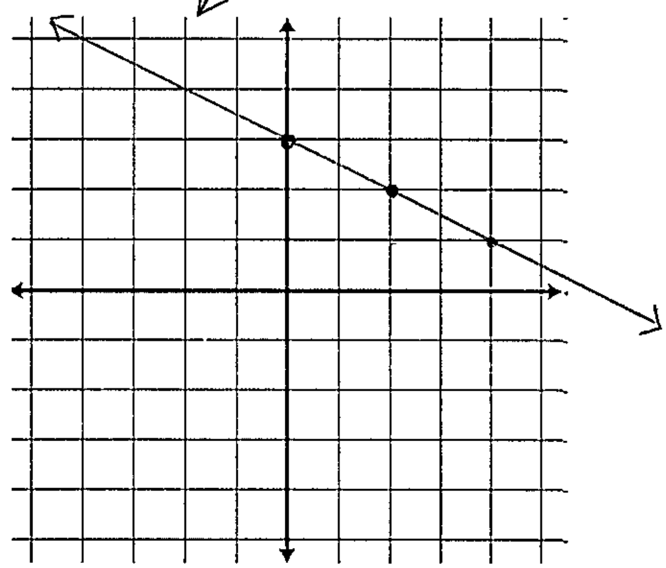
STEPS:

1. **Get "y" alone first!**
2. **Start by plotting the *y*-intercept.**
3. **Convert the slope to a fraction.**
4. **Use the "rise" & the "run" to plot the next few points.**
5. **Connect the points w/ a straight line.**



2. Graph the line $y = -\frac{1}{2}x + 3$. start

↳ slope



3. Graph the line $2y = -3x + 8$.

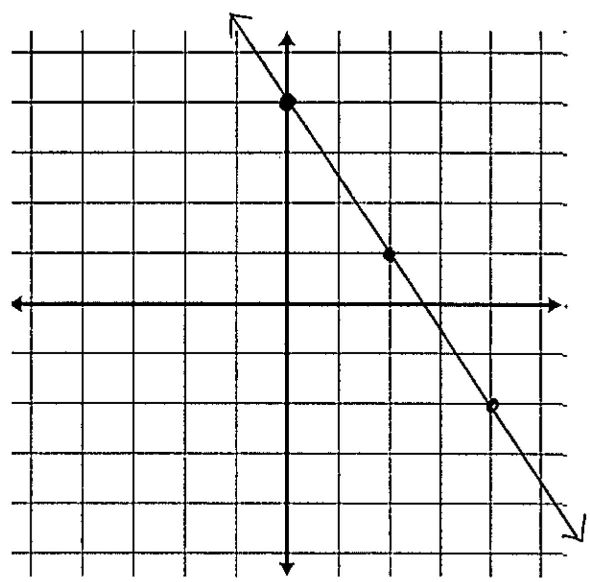
*get "y" alone

$$\frac{\cancel{2}y}{\cancel{2}} = \frac{-3x}{2} + \frac{8}{2}$$

$$y = \frac{-3}{2}x + 4$$

↓ slope

start

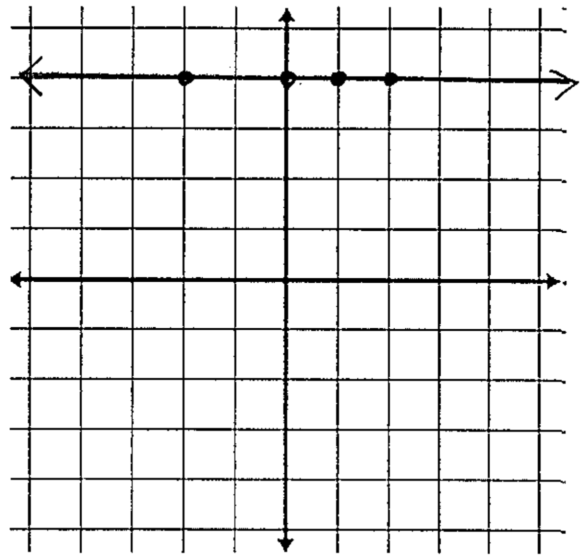


Weird Cases...

4. Graph the line $y = 4$.

x	y
-2	4
0	4
1	4
2	4

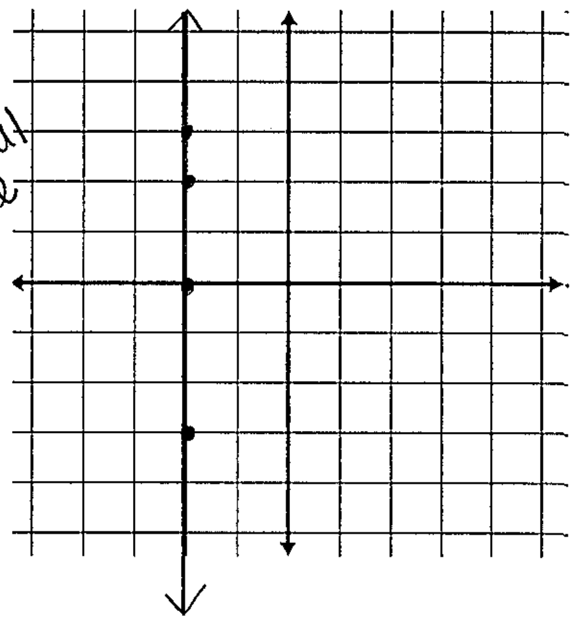
horizontal line



5. Graph the equation $x = -2$.

x	y
-2	-3
-2	0
-2	2
-2	3

vertical line



Looking at the weird cases...

Example 4 shows a horizontal line.

- Any equation in the form $y = \#$ will be a horizontal line.
- These lines have a slope of zero.

Example 5 shows a vertical line.

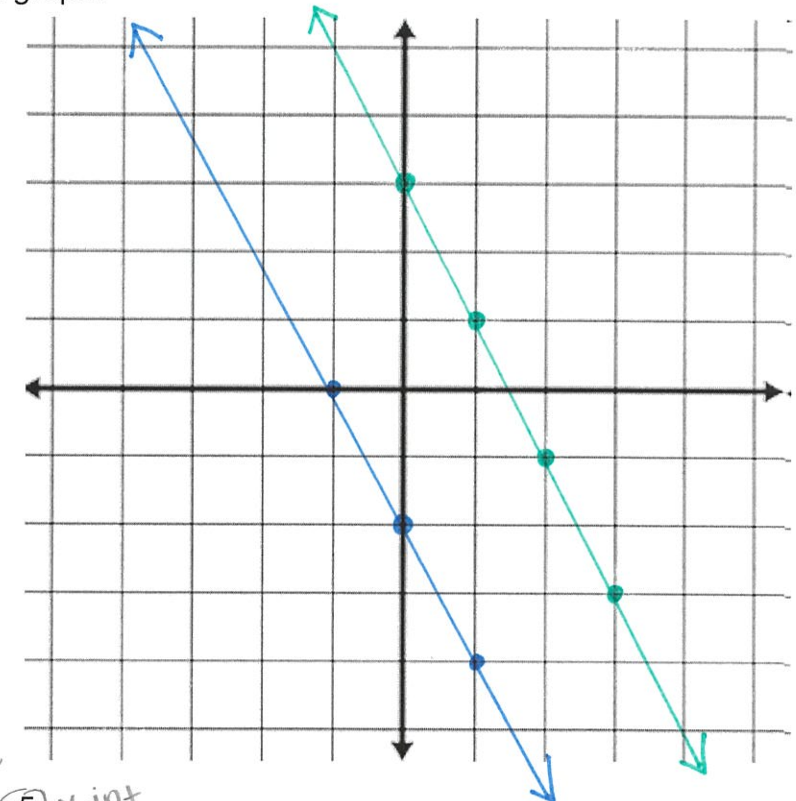
- Any equation in the form $x = \#$ will be a vertical line.
- These lines have an undefined slope.

6. Graph both lines on the same graph.

slope $-2/1$

- $y = -2x + 3$ start
- $y = -2x - 2$ start

slope $-2/1$



What do you notice???

same slope

The lines are //

7. Consider the equation $y = 3x - 5$. slope y-int

same slope = 3

A. Write an equation for a line that is parallel to this line and crosses the y-axis at 4.

b

$y = mx + b$

$\boxed{y = 3x + 4}$

B. Write an equation for a line that is perpendicular to this line and crosses the y-axis at -2.

b

opp/rec. slope = $3 \rightarrow -1/3$

$y = mx + b$

$\boxed{y = -1/3\, x + -2}$

Lesson 3-3: Linear Combination Situations

Vocabulary

Slope: steepness of a line

Slope = $\frac{\text{rise}}{\text{run}}$ OR $\frac{y_2 - y_1}{x_2 - x_1}$

Practice

1. In pro hockey, a win is worth 2 points, a tie is worth 1 point, and a loss is worth 0 points. Early in the season a team had 11 points.

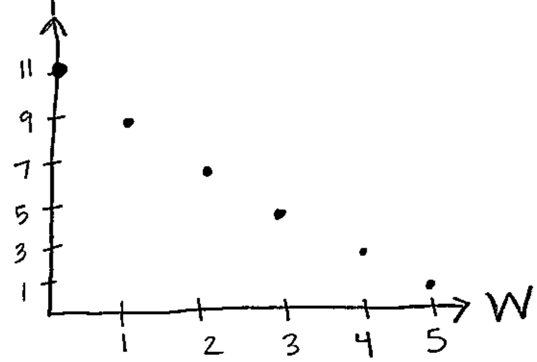
   A. Write an equation to express the relationship among the number of wins W, the number of ties T, and the number of losses L.

   $$2W + 1T + \cancel{0L} = 11$$

   $$\boxed{2W + 1T = 11}$$

   B. Make a table and graph of all possible (W, T) that could occur.

W	T
0	11
1	9
2	7
3	5
4	3
5	1



2. A chemist mixes *x* ounces of a 20% acid solution with *y* ounces of a 30% solution. The final mixture contains 9 ounces of acid.

   A. Write an equation relation *x*, *y*, and the total number of ounces of acid.

   $$0.2x + 0.3y = 9$$

   B. How many ounces of the 30% acid solution must be added to 2.7oz of the 20% solution to get 9oz of acid in the final mixture?

   ✓ x

   $$0.2(2.7) + 0.3y = 9$$

   $$.54 + 0.3y = 9$$

   $$-.54 \qquad -.54$$

   $$\frac{0.3y}{0.3} = \frac{8.46}{0.3}$$

   $$y = \boxed{28.2\text{ oz}}$$

$x_1 \; y_1 \qquad x_2 \; y_2$

3. Find the slope given two points (-2, 3) and (4, 15).

$\downarrow$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 3}{4 - -2} = \frac{12}{6} = \boxed{2}$$

Lesson: Intercepts

We already know how to find the y-intercept by looking at an equation in slope-intercept form. Now, we are going to learn how to find the x & y intercepts from ANY form.

Vocabulary

Intercept: the specific point where a line crosses the x or y axis.

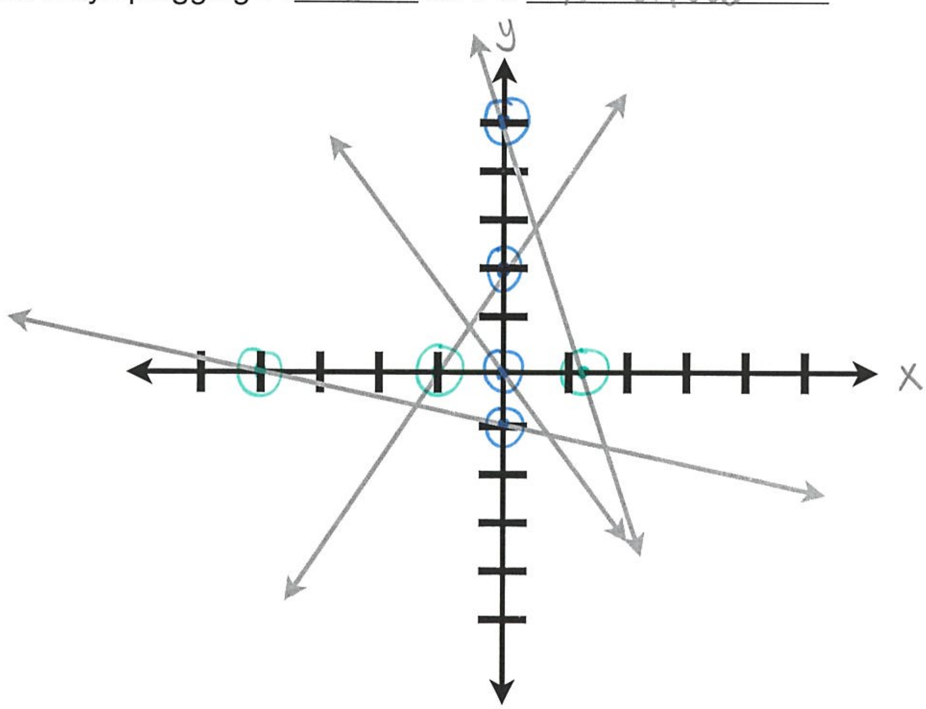
*they are very useful in graphing, solving, and interpreting real-life situations!

x-intercept: where a line crosses the x-axis; the y-coordinate is always zero

Find it by...plugging in 0 for the y-value

y-intercept: where a line crosses the y-axis; the x-coordinate is always zero

Find it by...plugging in 0 for the x-value



Practice

Find the x & y intercepts of each equation.

1. $y = 2x - 3$

x-intercept	y-intercept
$y = 2x - 3$	$y = 2x - 3$
$0 = 2x \cancel{-3}$	$y = 2 \cdot 0 - 3$
$+3 \qquad \cancel{+3}$	$y = -3$
$\frac{3}{2} = \frac{\cancel{2}x}{\cancel{2}}$	So, $\boxed{(0, -3)}$
$1.5 = x$	
So, $\boxed{(1.5, 0)}$	

2. $2x + 4y = 32$

x-intercept	y-intercept
$2x + 4y = 32$	$2x + 4y = 32$
$2x + 4 \cdot 0 = 32$	$2 \cdot 0 + 4y = 32$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{32}{2}$	$\frac{\cancel{4}y}{\cancel{4}} = \frac{32}{4}$
$x = 16$	$y = 8$
So, $\boxed{(16, 0)}$	So, $\boxed{(0, 8)}$

3. $y = -2$

horizontal line!

x-intercept	y-intercept
none!	$y = -2$
	So, $\boxed{(0, -2)}$

Lesson 3-4: Graphing in Standard Form

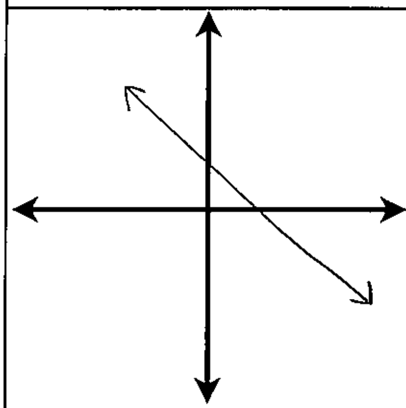
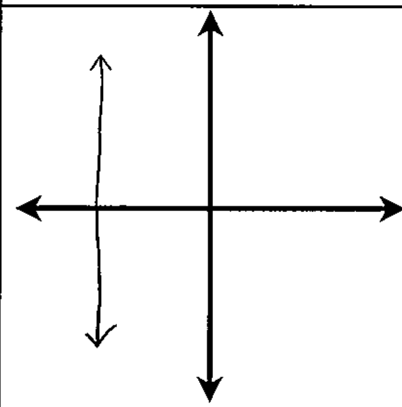
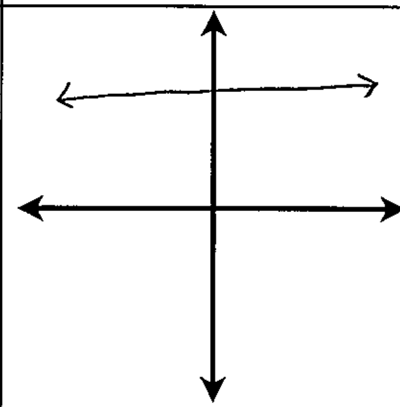
Vocabulary

Remember, we already learned about slope-intercept form of a linear equation, but that is not the only form. We also have...

Standard Form: $Ax + By = C$

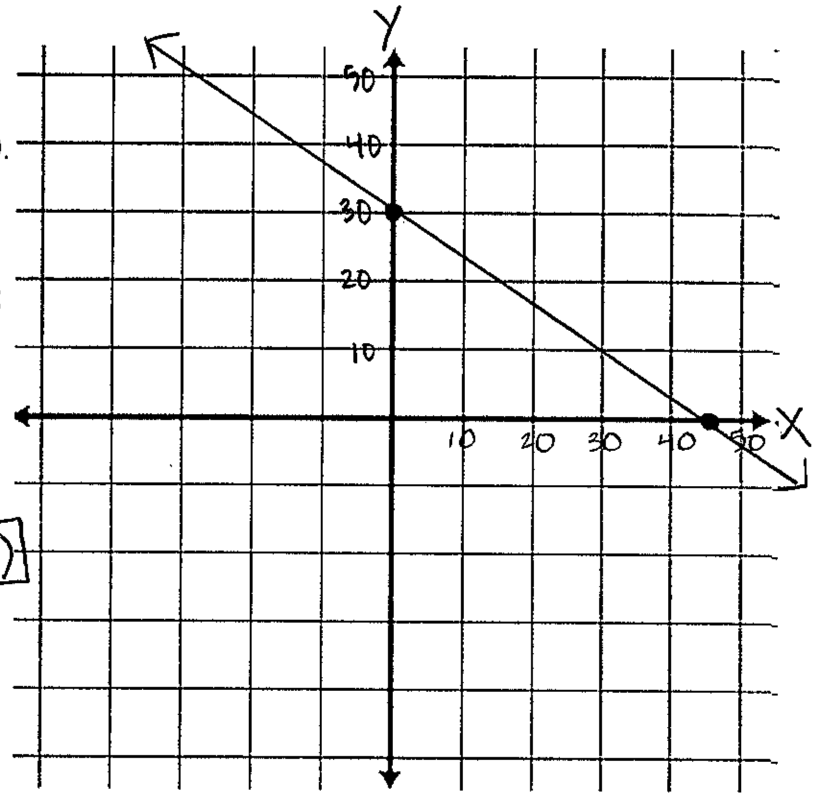
- A, B, and C are all integers (no fractions or decimals!)
- It is easiest to graph from standard form using the intercepts

TYPES OF LINES

Oblique Line	Vertical Line	Horizontal Line
		
Looks like... $y = mx + b$ or... $Ax + By = C$ Slope = $-A/B$ or m	Looks like... $x = \#$ Slope = undefined	Looks like... $y = \#$ Slope = zero

Practice

1. Using the equation $0.2x + 0.3y = 9$.



STEPS:
1. **Find the intercepts.**
2. **Plot the intercepts & connect with a straight line!**

***x*-intercept:** 0 in for "y"

$0.2x + 0.3 \cdot 0 = 9$

$\frac{\cancel{0.2}x}{\cancel{0.2}} = \frac{9}{0.2}$ So, $\boxed{(45,0)}$

$x = 45$

***y*-intercept:** 0 in for "x"

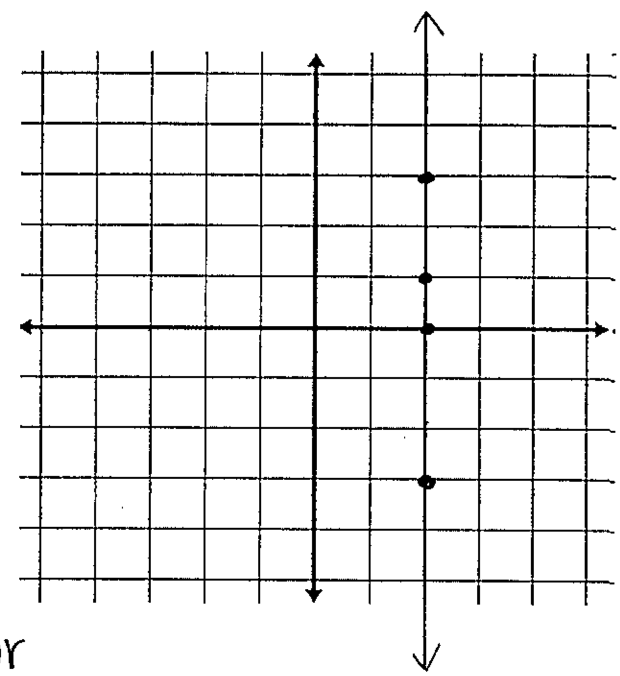
$0.2 \cdot 0 + 0.3y = 9$

$\frac{\cancel{0.3}y}{\cancel{0.3}} = \frac{9}{0.3}$ So, $\boxed{(0,30)}$

$y = 30$

2. A. Graph the line $x = 2$.

x	y
2	-3
2	0
2	1
2	3



B. Find the slope of the line.

Two points

$\frac{\text{rise}}{\text{run}} = \frac{1}{0} = \text{error}$

$\boxed{\text{Slope = undefined}}$

Lesson 3-5: Writing the Equation for a Line

Vocabulary

When possible, use slope-int. form! You only need two pieces of information...

$$y = mx + b$$

m: slope; b: y-int

Practice

1. slope = -3, *y*-intercept = 2 — Equation: $y = -3x + 2$
2. slope = 1/2, *y*-intercept = -5 — Equation: $y = \frac{1}{2}x - 5$
3. slope = 0, *y*-intercept = -3.5 — Equation: $y = 0x - 3.5$ or $y = -3.5$
4. slope = -4, *y*-intercept = 0 — Equation: $y = -4x + 0$ or $y = -4x$

BUT WAIT!!! What if you don't know the slope?!?! Then... find it!

Practice

5. Contains points $(-2, 3)$ & $(6, -1)$; *y*-intercept = 2.

$x_1\ y_1$, $x_2\ y_2$; $2 \leftarrow b$

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-3}{6-3} = \frac{-4}{3} \leftarrow m$$

Equation: $y = -\frac{4}{3}x + 2$

6. Contains points $(7, 3)$; *y*-intercept = 5.

$x_1\ y_1$; $5 \leftarrow b$

point $(0, 5)$ — $x_2\ y_2$

$$\frac{5-3}{0-7} = \frac{2}{-7} \leftarrow m$$

Equation: $y = -\frac{2}{7}x + 5$

7. Contains points $(-2, 4)$ & $\boxed{(0, 12)}$. y-int = 12 ← b

$x_1\ y_1$, $x_2\ y_2$

$$\frac{12-4}{0--2} = \frac{8}{2} = 4 \leftarrow m$$

Equation: $y = 4x + 12$

BUT WAIT!!! What if you don't know the y-intercept?!?! That's a little different...

When you don't know the *y*-intercept, you can use a new form called point-slope form.

slope

$$y - y_1 = m(x - x_1)$$

coordinates of ANY point

We can work from point-slope form to find slope-int. form!

Practice

8. Find the equation of a line with slope = 3, passing through point (-2, 10).

m $\quad x_1, y_1$

$$y - y_1 = m(x - x_1)$$

$$y - 10 = 3(x + 2)$$

$$y - \cancel{10} = 3x + 6$$

$$+\cancel{10} \qquad +10$$

$$\boxed{y = 3x + 16}$$ ← slope-int. form

9. Find the equation of a line through point (3, 5) and (6, -1).

$x_1, y_1 \quad x_2\ y_2$

① Find slope, since its not given

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{6-3} = \frac{-6}{3} = -2 \quad m$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - 3)$$

$$y - 5 = -2x + 6$$

$$+5 \qquad +5$$

$$\boxed{y = -2x + 11}$$

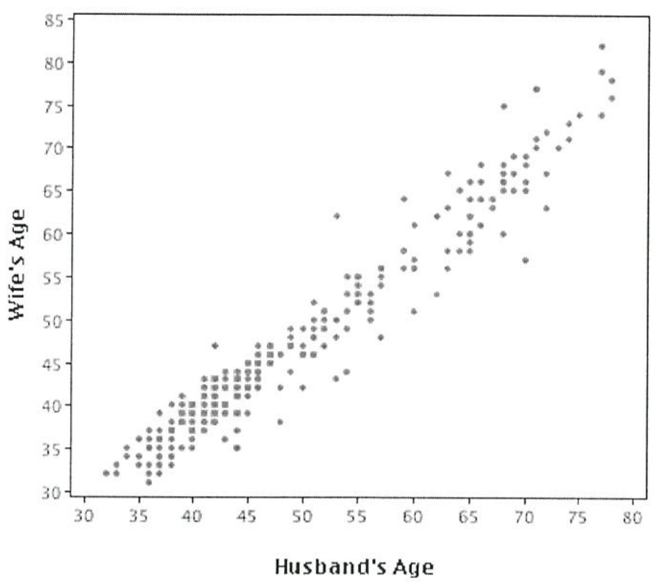
Lesson 3-6: Linear Regression

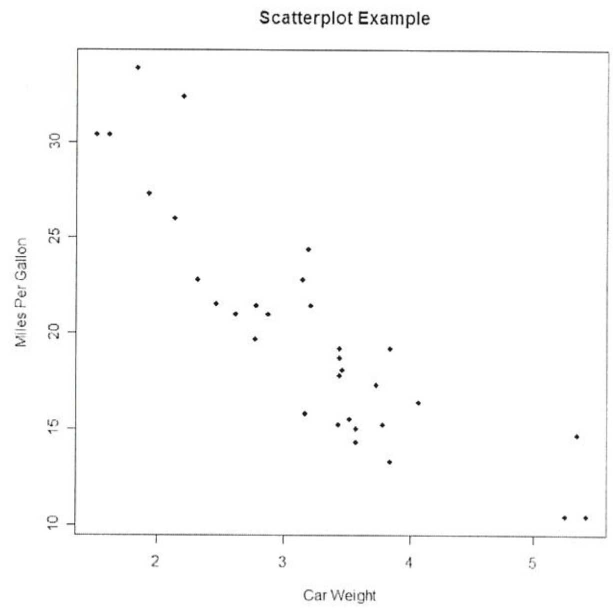
Vocabulary

Scatterplot: a way to display a set of data as discrete data points

- Data is numerical
- Data is usually collected from an experiment of some type of trial

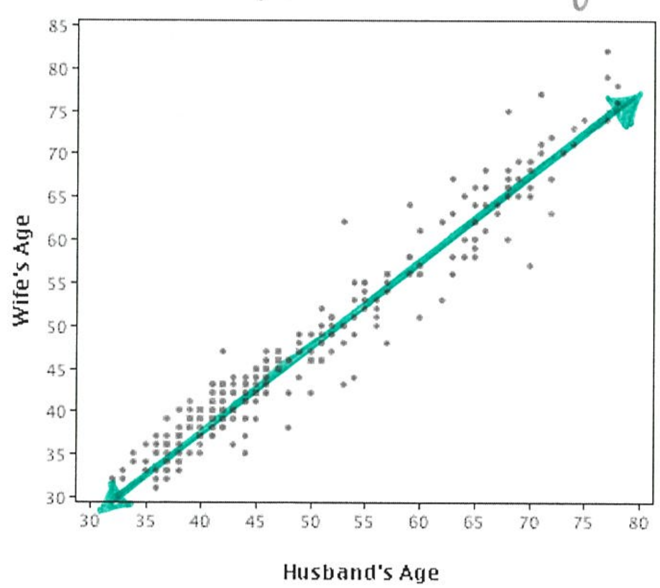
Examples





Line of Best-Fit/Line of Regression: a line that follows the path of data approximately.

Example



***useful for making predictions with data**

Correlation: a measure of the relationship between the variables

Positive Correlation means that as one quantity increases, the other increases as well.

Negative Correlation means that as one quantity increases, the other decreases.

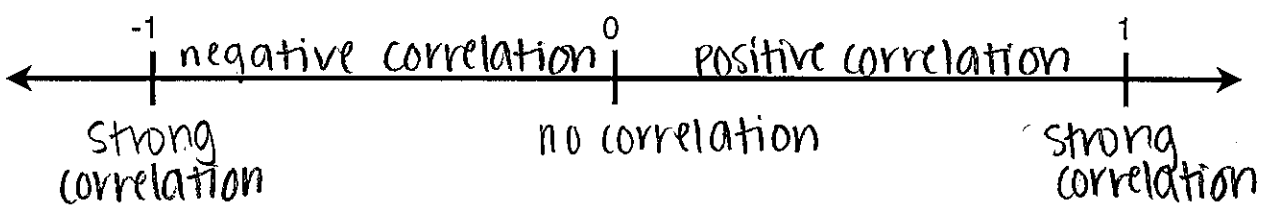
***NOTE: Correlation does NOT imply causation!!! Just because two variables are correlated, does NOT mean that one is causing the other!

Correlation Coefficient (*r*): gives a numerical value for correlation (letter "r" on your calculator)

If *r* is positive, the correlation is + & the slope of the line is +.

If *r* is negative, the correlation is − & the slope of the line is −.

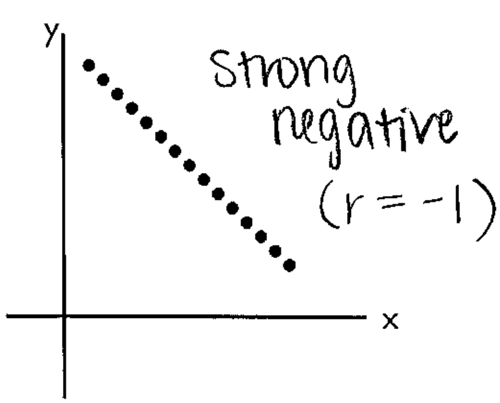
The **absolute value of *r*** indicates the strength of the correlation or linear relationship. The closer the magnitude of *r* is to 1, the stronger the linear relationship.



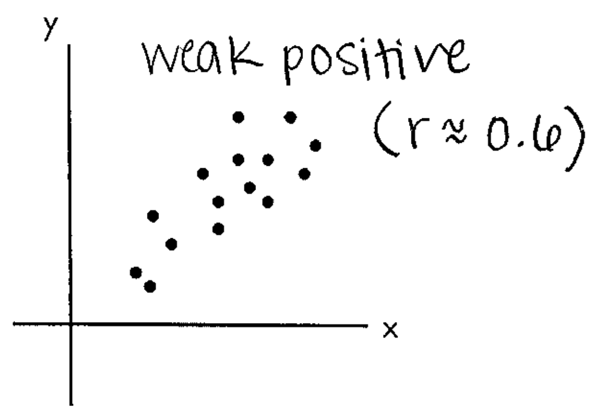
Practice

1. Identify the type of correlation for each data set. Give an approximate value for *r*.

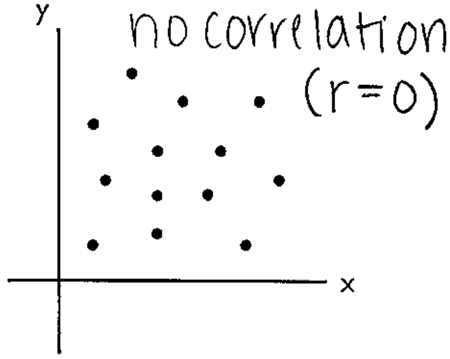
A.



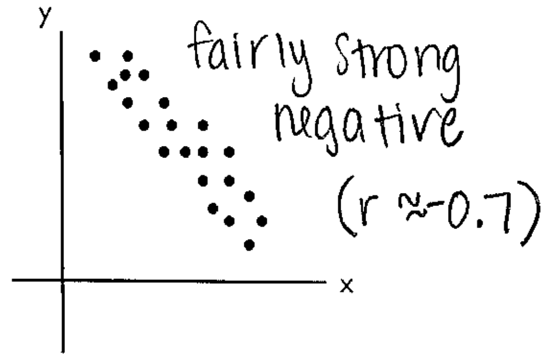
B.



C.



D.



2. Find the line of best-fit that represents the data below. Interpret the slope in context.

# of Absences	0	1	2	3	4	5	6	7	8
Avg. Grade	92	90	83	76	72	68	60	54	50

① Stat → Edit... → L1 & L2 (enter data)

② Stat → Calc → LinReg(ax+b) → Enter

$y = -5.5$

$b = 93.7$

$r = -.99$ ← strong negative correlation

$$\boxed{y = -5.5x + 93.7}$$

For each absence, a students grade drops 5.5 points.

3. Find the line of best-fit that represents the data below. Interpret the slope in context.

# of siblings	0	1	2	3	4
Avg. Grade	78	84	86	76	80

$r = -.152$ ← weak neg. correlation

$$\boxed{y = -0.4x + 81.6}$$

For each sibling, a students grade drops 0.4 points

* Bad assumption!!!