Name: KEY!

Hour: ______

CHAPTER 11:

POLYNOMIALS

It is nice to be important. But, it is more important to be nice.

Lesson 11-1: Introduction to Polynomials

Vocabulary

Polynomial: a polynomial is an expression made up of variables & constants that are combined using addition, subtraction, & mult. w/ whole # powers.

Example: $2x^4 - 3x^3 + x^2 + 5x - 6$

Degree: the largest exponent on the variable.

Standard Form: polynomial written in descending order of exponents

Leading Coefficient: the coefficient on the 1st term in the polynomial (when written in standard form)

Term: the "pieces" of the polynomial separated by + or - signs.

Special Polynomials		
Linear	polynomial in first degree	$2x + 3$
Quadratic	polynomial in second degree	$2x^2 + 4x - 6$
Cubic	polynomial in third degree	$5x^3 + 6$
Quartic	polynomial in fourth degree	$8x^4 + 2x^3 + x$

Practice

1. Consider the following polynomial: $-2m^4 + 3m + 6m^5 - 5m^2 + 1$

   a) Write the polynomial in standard form.

   $6m^5 - 2m^4 - 5m^2 + 3m + 1$

   b) Identify the leading coefficient.

   6

   c) Give the degree of the polynomial.

   5

   d) State the number of terms.

   5 "pieces"

2. Starting the summer after her senior year in high school, Mrs. Merritt worked to save money for a new car. At the end of each summer, she deposited her money into an account with an interest rate of r%. She planned to get a new car after her fourth year of college.

   $1 + r$

after senior year	\$1500
after 1st year college	\$2200
after 2nd year college	\$2100
after 3rd year college	\$3000
after 4th year college	\$3300

   a) Write a polynomial expression representing the amount she will have saved by the end of her fourth year of college.

   $1500(1+r)^4 + 2200(1+r)^3 + 2100(1+r)^2 + 3000(1+r) + 3300$

   b) If she earns 6% interest each year, calculate how much she will have saved.

   .06

   $1500(1.06)^4 + 2200(1.06)^3 + 2100(1.06)^2 + 3000(1.06) + 3300 = \boxed{\$13,353.51}$

Lesson: Multiplying Polynomials

Remember...

When multiplying powers, Multiply the coefficients, but add the exponents!

Example: $(3x^2)(4x^5) =$ $12x^7$

The key to multiplying polynomials is…

distributing everything!

Practice

1) $(x^2 + 3x - 1)(2x^3 - 4x)$

$\cancel{2x^5} - 4x^3 + \cancel{6x^4} - 12x^2 - 2x^3 + 4x$

$\boxed{2x^5 + 6x^4 - 6x^3 - 12x^2 + 4x}$

2) $(4r^3 - 3p^2r^2)(8pr^5 + 6p^3r^2 - 5r)$

$32r^8p + 24r^5p^3 - 20r^4 - 24r^7p^3 - 18r^4p^5 + 15r^3p^2$

If you're confused, try using a box to keep terms straight.

3) $(6a^3b^2 - a^2b + 2b^3)(7ab + 5a^2b + a^4)$

	$7ab$	$5a^2b$	a^4
$6a^3b^2$	$42a^4b^3$	$30a^5b^3$	$6a^7b$
$-a^2b$	$-7a^3b^2$	$-5a^4b^2$	$-a^6b$
$2b^3$	$14ab^4$	$10a^2b^4$	$2a^4b^3$

$42a^4b^3 + 30a^5b^3 + 6a^7b^2$
$-7a^3b^2 - 5a^4b^2 - a^6b$
$+ 14ab^4 + 10a^2b^4 + 2a^4b^3$

Lesson 11-2: Applications & Problem Solving w/ Polynomials

Vocabulary

Monomial: a polynomial w/ one term, ex: $5x$ or $3x^4$

Binomial: a polynomial w/ 2 terms, ex: $5x+2$ or $8x^3-2x$

Trinomial: a polynomial w/ 3 terms, ex: $5x^2-3x+4$ or $8x^3+4x+1$

**A quick note about the "degree" of a polynomial…*

If a term has more than one exponent, the total degree is the sum of the exponents

The degree of a polynomial is the largest of its terms!

Practice

Classify each polynomial, then give its degree.

1. $\overset{12}{3m^7n^5} + \overset{13}{6m^4n^9} - \overset{9}{m^3n^6}$

trinomial

degree: 13

2. $-\overset{4}{2x^2y^2} + \overset{6}{6xy^5}$

binomial

degree: 6

3. Give the dimensions of the rectangle and find the area.

$2k+6s+7r$

	2k	6s	7r
3k	$6k^2$	18ks	21kr
12	24k	72s	84r

$3k+12$

Dimensions: $3k+12$ & $2k+6s+7r$

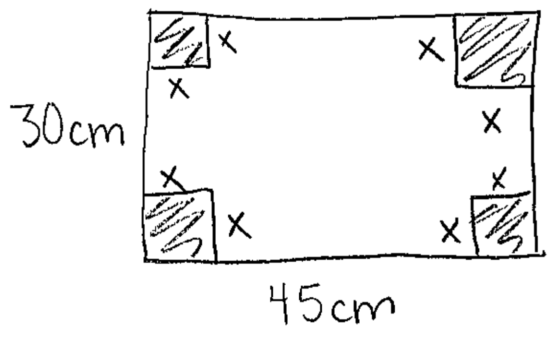
Area: $(3k+12)(2k+6s+7r)$

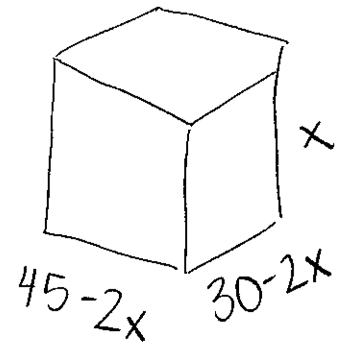
$$\boxed{= 6k^2 + 18ks + 21kr + 24k + 72s + 84r}$$

4. An open box is folded from a sheet of cardboard so that it is 30cm by 45cm by removing squares of side length *x* from each corner.

Write an expression to represent the total volume of the box.

$l \cdot w \cdot h$





$$V = (45-2x)(30-2x)(x)$$

$$= (1350 - 90x - 60x + 4x^2)\, x$$

$$= (1350 - 150x + 4x^2)(x)$$

$$= 1350x - 150x^2 + 4x^3$$

$$\boxed{4x^3 - 150x^2 + 1350x}$$

Lesson: Area & Perimeter Practice w/ Polynomials

Vocabulary

Perimeter: the distance around the outside of a figure

Area: the amount of space inside a 2D figure

Remember your Geometry area formulas…

Triangle: $A = \frac{1}{2} \cdot b \cdot h$

Square/Rectangle: $A = b \cdot h$

Circle: $A = \pi \cdot r^2$ & $C = 2\pi r$

Practice

Find the perimeter and area of each figure.

1.

$P = 5g + 4 + 2g - 3 + 5g + 4 + 2g - 3$

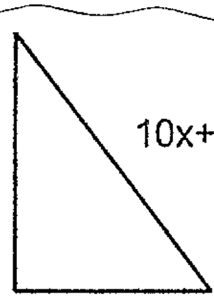
$\boxed{P = 14g + 2}$

$A = (5g+4)(2g-3)$

$A = 10g^2 - 15g + 8g - 12$

$\boxed{A = 10g^2 - 7g - 12}$

2.



$P = 10x + 12 + 4x^2 + 8x + 2x^2 - 3x + 1$

$\boxed{P = 6x^2 + 15x + 13}$

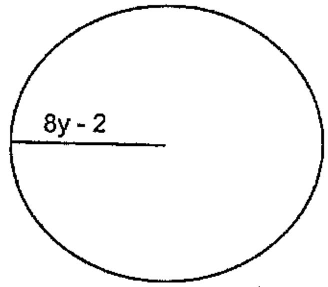
$A = \frac{1}{2}(4x^2 + 8x)(2x^2 - 3x + 1)$

$A = (2x^2 + 4x)(2x^2 - 3x + 1)$

$A = 4x^4 - 6x^3 + 2x^2 + 8x^3 - 12x^2 + 4x$

$\boxed{A = 4x^4 + 2x^3 - 10x^2 + 4x}$

3.



$C = 2 \cdot \pi \cdot (8y - 2)$

$\boxed{C = 16\pi y - 4\pi}$

$A = \pi(8y-2)^2$

$A = \pi(8y-2)(8y-2)$

$A = \pi(64y^2 - 16y - 16y + 4)$

$A = \pi(64y^2 - 32y + 4)$

$A = 64y^2\pi - 32y\pi + 4\pi$

Lesson: Factoring with GCF

Vocabulary

GCF: the largest # that divides into ALL terms

Examples:

4 & 18	12 & 40	27 & 63	30, 84, & 210
GCF = 2	GCF = 4	GCF = 9	GCF = 6

Factoring: breaking up a polynomial into simpler terms (factors)

*polynomials that cannot be factored are called prime.

Remember we already know how to FOIL...

Examples:

FOIL

$$(x+2)(x+3) \quad x^2+2x+3x+6 = \quad x^2+5x+6$$

FACTOR

So, to factor a trinomial you will be starting with something that looks like this: x^2+bx+c and ending with something that looks like this: (x +/- _____)(x +/- _____).

Using the above example, check this out:

$$(x+2)(x+3) = x^2+5x+6$$

2+3
add together
mult. together
2·3

Practice

1. $x^2+12x+35$

$(x+7)(x+5)$

$7+5=12$ ✓
$7\cdot5=35$ ✓

2. x^2+3x+2

$(x+1)(x+2)$

$1+2=3$ ✓
$1\cdot2=2$ ✓

3. $x^2+10x+24$

$(x+6)(x+4)$

$6+4=10$ ✓
$6\cdot4=24$ ✓

If "c" is pos., the two #'s must both be pos. or both be neg.

That works out really well, but what if the middle "b" number happens to be **negative**??? That would like like this... $x^2 - bx + c$.

If "b" is neg. & "c" is pos, then the two #'s must be neg!

Example: $x^2 - 8x + 15$

$(x-3)(x-5)$

$-3 + -5 = -8$ ✓

$-3 \cdot -5 = 15$ ✓

Practice

4. $x^2 - 7x + 10$

$(x-2)(x-5)$

$-2 + -5 = -7$ ✓

$-2 \cdot -5 = 10$ ✓

5. $x^2 - 2x + 1$

$(x-1)(x-1)$

$-1 + -1 = -2$ ✓

$-1 \cdot -1 = 1$ ✓

That works out really well, but what if the last number "c" happens to be **negative**??? That would like like this... $x^2 + bx - c$

one # must be pos & one must be neg

If "b" is pos, the larger # must be pos.
If "b" is neg, the larger # must be neg

Example: $x^2 + 4x - 21$

$(x+7)(x-3)$

$7 + -3 = 4$ ✓

$7 \cdot -3 = -21$ ✓

Practice

6. $x^2 + x - 2$

$(x+2)(x-1)$

$2 + -1 = 1$ ✓

$2 \cdot -1 = -2$ ✓

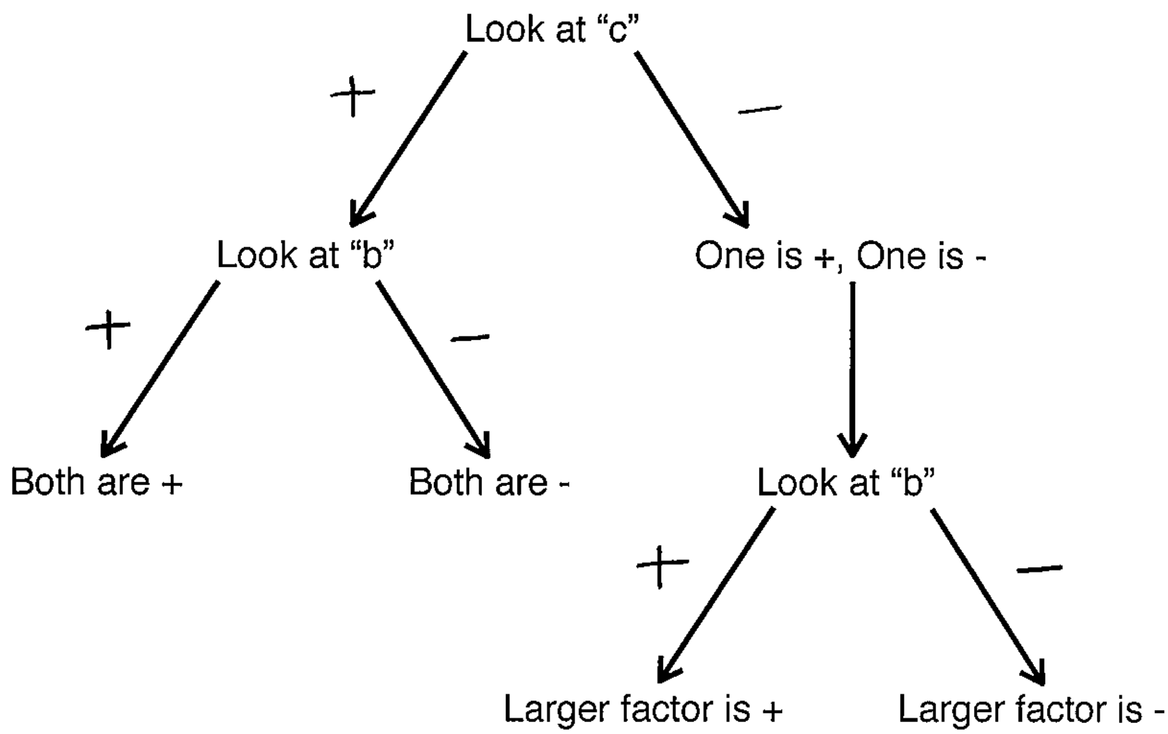
7. $x^2 - 3x - 28$

$(x+4)(x-7)$

$4 + -7 = -3$ ✓

$4 \cdot -7 = -28$ ✓

How to Pick Your Signs:



Practice

1. $x^2+11x+24$

$(x+8)(x+3)$

$8+3=11$ ✓

$8\cdot 3=24$ ✓

2. x^2-6x+8

$(x-2)(x-4)$

$-2+-4=-6$ ✓

$-2\cdot -4=8$ ✓

3. $x^2-2x+-15$

$(x+3)(x-5)$

$3+-5=-2$ ✓

$3\cdot -5=-15$ ✓

Factor Practice ~ Mixed

Factor using GCF.

1. $\underline{36}x^3 + \underline{24}x^2 - \underline{48}x - \underline{12}$

GCF = 12

$12(3x^3 + 2x^2 - 4x - 1)$

2. $\underline{21}m^4n^2 - \underline{14}m^5n^4 + \underline{56}m^2n^2$

GCF = 7, m^2 & n^2

$7m^2n^2(3m^2 - 2m^3n^2 + 8)$

Factor each trinomial.

3. $x^2 - 3x + 2$

$(x-2)(x-1)$

$-2 + -1 = -3$ ✓

$-2 \cdot -1 = 2$ ✓

4. $x^2 - 8x - 48$

$(x+4)(x-\underline{12})$

$4 + -12 = -8$ ✓

$4 \cdot -12 = -48$ ✓

Factor using GCF, then factor the trinomial.

5. $3x^3 + 3x^2 - 36x$ GCF = 3 & x

$3x\,\underbrace{(x^2 + x - 12)}$

$\boxed{3x(x+4)(x-3)}$

$4 + -3 = 1$ ✓

$4 \cdot -3 = -12$ ✓

Special Cases of Factoring

Factor $x^2 - 25 \rightarrow x^2 + 0x - 25 \rightarrow (x+5)(x-5)$

$5 + -5 = 0$ ✓
$5 \cdot -5 = -25$ ✓

Difference of Squares	
$a^2 - b^2$	$(a+b)(a-b)$

Practice

1. $x^2 - 36$

$(x+6)(x-6)$

2. $x^2 - 16y^2$

$(x+4y)(x-4y)$

3. $9x^2 - 49$

$(3x+7)(3x-7)$

4. $16x^4 - 81$

$(4x^2+9)(4x^2-9)$

Difference of Cubes	
$a^3 - b^3$	$(a-b)(a^2+ab+b^2)$

$\sqrt[3]{x^3} = x$ $\sqrt[3]{27} = 3$

5. $x^3 - 27$

$(x-3)(x^2+3x+9)$

$\sqrt[3]{8x^3} = 2x$ $\sqrt[3]{64} = 4$

6. $8x^3 - 64$

$(2x-4)(4x^2+8x+16)$

$\sqrt[3]{125x^3} = 5x$ $\sqrt[3]{1} = 1$

7. $125x^3 - 1$

$(5x-1)(25x^2+5x+1$

Sum of Cubes	
$a^3 + b^3$	$(a+b)(a^2-ab+b^2)$

$\sqrt[3]{27x^3} = 3x$ $\sqrt[3]{1} = 1$

8. $27x^3 + 1$

$(3x+1)(9x^2-3x+1)$

$\sqrt[3]{8x^3} = 2x$ $\sqrt[3]{8y^3} = 2y$

9. $8x^3 + 8y^3$

$(2x+2y)(4x^2-4xy+4y^2)$