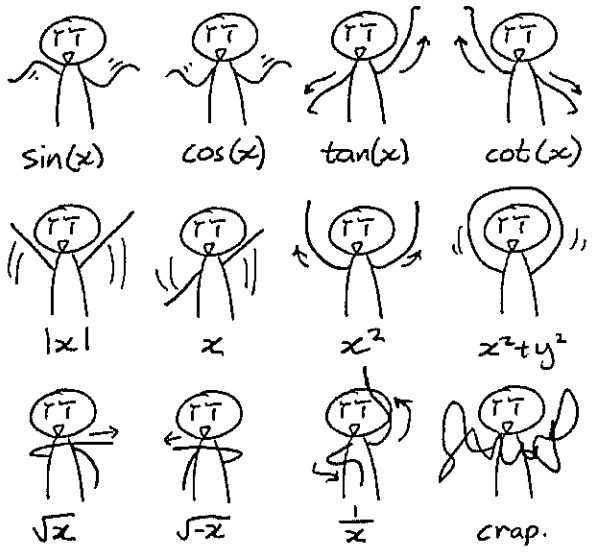
Name: KEY!

Hour: _____

Chapter 10:

Trigonometry

Beautiful Dance Moves



Lesson 10-1: Trigonometry Ratios

In this chapter, your calculator MUST be set for degrees, not radians. To change this setting: (1) Mode, (2) Select "Degree" on the third row

Vocabulary

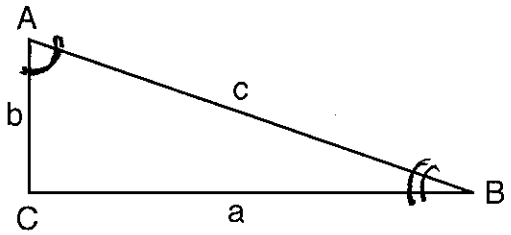
Trigonometry: the study of right triangles

Hypotenuse: the side across from the 90°

Rounding Information - when working in trigonometry, round to 4 decimal places ***while* you are working through a problem, then at the very end you can round your final answer to the nearest tenth (one decimal).

Sine: the ratio of the **o**pposite side to the **h**ypotenuse

Example:



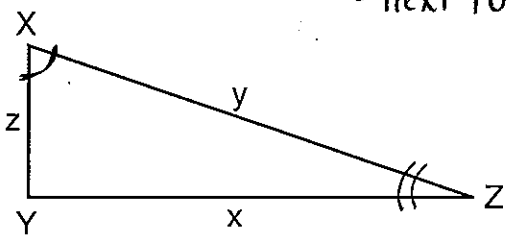
$\sin A = \frac{opp}{hyp} = \boxed{\frac{a}{c}}$

$\sin B = \frac{opp}{hyp} = \boxed{\frac{b}{c}}$

Cosine: the ratio of the **a**djacent side to the **h**ypotenuse

↑ next to

Example:

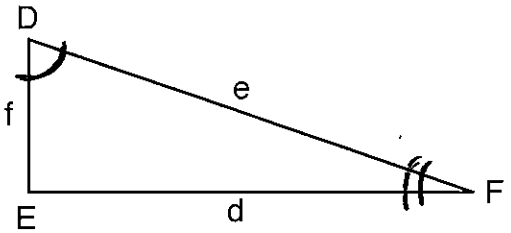


$\cos X = \frac{adj}{hyp} = \boxed{\frac{z}{y}}$

$\cos Z = \frac{adj}{hyp} = \boxed{\frac{x}{y}}$

Tangent: the ratio of the **o**pposite side to the **a**djacent side

Example:



$\tan D = \frac{opp}{adj} = \boxed{\frac{d}{f}}$

$\tan F = \frac{opp}{adj} = \boxed{\frac{f}{d}}$

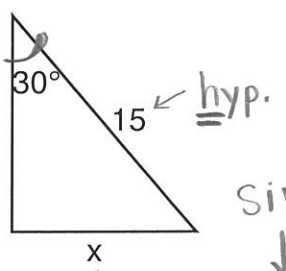
SOH - CAH - TOA

$\sin\theta = \frac{opp}{hyp}$ $\cos\theta = \frac{adj}{hyp}$ $\tan\theta = \frac{opp}{adj}$

*"θ" means the angle!

Practice

1. Find the value of *x*.



hyp.

opp.

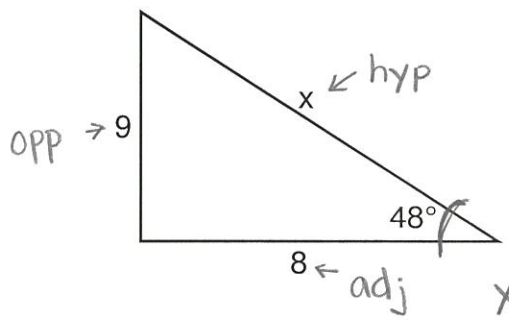
sine

$\sin\theta = \frac{opp}{hyp}$

$15 \cdot \sin 30° = \frac{x}{\cancel{15}} \cdot \cancel{15}$

$\boxed{7.5 = x}$

2. Find the value of *x*.



hyp

opp → 9

adj

sine or cosine

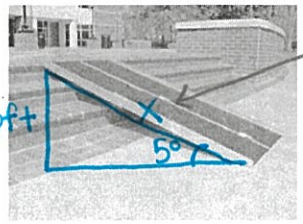
$\cos\theta = \frac{adj}{hyp}$

$x \cdot \cos 48° = \frac{8}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \cdot \cancel{\cos 48}}{\cancel{\cos 48}} = \frac{8}{\cos 48}$

$\boxed{x = 11.96}$

3. The building code in one city specifies that ramps must form an angle with the ground no greater than 5°. A porch is 6 ft high. If a contractor wants to build a handicap-accessible ramp to reach this porch, what is the shortest this ramp can be?



hyp

opp

sine

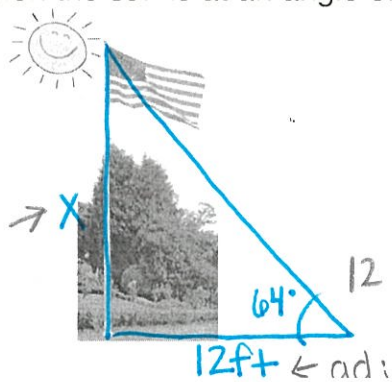
$\sin\theta = \frac{opp}{hyp}$

$x \cdot \sin 5° = \frac{6}{\cancel{x}} \cdot \cancel{x}$

$\frac{x \cdot \cancel{\sin 5°}}{\cancel{\sin 5°}} = \frac{6}{\sin 5°}$

$\boxed{x = 68.8 \text{ ft}}$

4. A flagpole casts a 12 ft shadow when the sun is at an angle of 64° with the ground. How tall is the flagpole?



opp. → x

adj

tangent

$\tan\theta = \frac{opp}{adj}$

$12 \cdot \tan 64° = \frac{x}{\cancel{12}} \cdot \cancel{12}$

$\boxed{24.6 = x}$

Lesson 10-2: Inverse Trigonometry

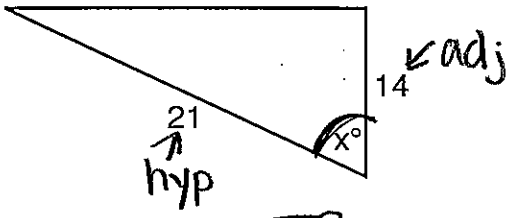
Vocabulary

Inverse Trig Functions: allow us to find missing angle measures.

Practice

SOH-CAH-TOA

1. Find the value of *x*.



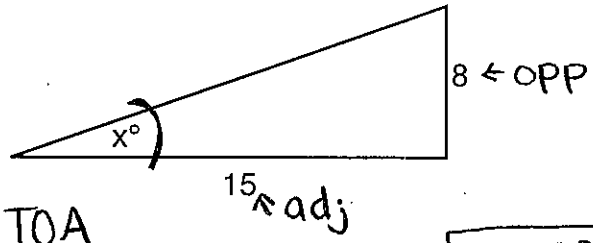
CAH

$\cos x° = \frac{14}{21}$

$x° = \cos^{-1}\left(\frac{14}{21}\right)$

$\boxed{x = 48°}$

2. Find the value of *x*.



TOA

$\tan x° = \frac{8}{15}$

$x° = \tan^{-1}\left(\frac{8}{15}\right)$

$\boxed{x = 28°}$

3. Find the measure of each angle. Round to the nearest degree.

a. $\cos\theta = 0.914$

$\theta = \cos^{-1} .914$

$\boxed{\theta = 24°}$

b. $\sin\theta = 0.829$

$\theta = \sin^{-1} .829$

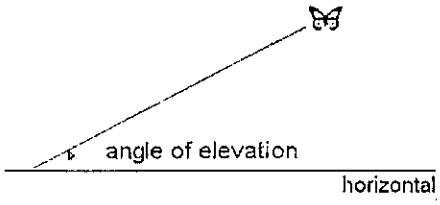
$\boxed{\theta = 56°}$

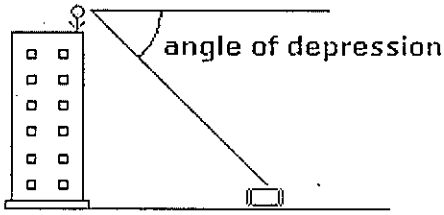
c. $\tan\theta = 4.705$

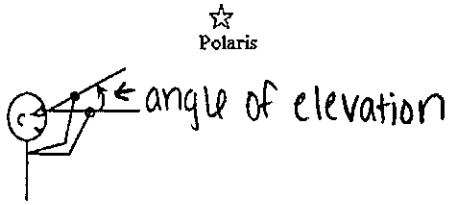
$\theta = \tan^{-1} 4.705$

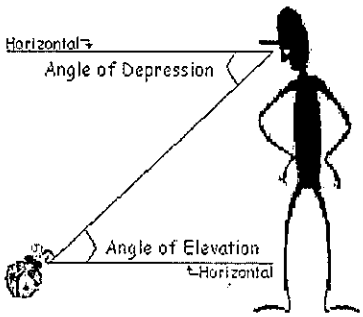
$\boxed{\theta = 78°}$

Angle of Elevation vs. Angle of Depression

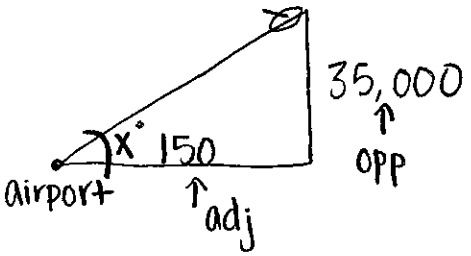








4. An airplane pilot is preparing to take off. According to air traffic control, he must reach an altitude of 35,000 ft. by the time he is 150 miles from the airport to avoid a collision. At what angle should the pilot ascend?



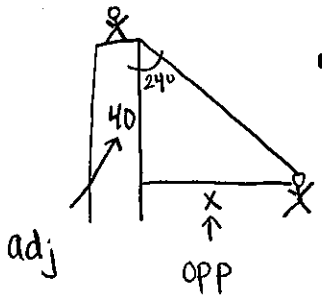
TOA

$$\tan x^\circ = \frac{35{,}000}{150}$$

$$x^\circ = \tan^{-1}\left(\frac{35{,}000}{150}\right)$$

$$\boxed{x = 89.8^\circ}$$

5. A construction worker is on top of a 40 ft tall building looking down at his assistant at a 24° angle. How far away from the building is his assistant?



$$40 \cdot \tan 24^\circ = \frac{x}{\cancel{40}} \cdot \cancel{40}$$

$$\boxed{17.8 = x}$$

Lesson 10-3: Trigonometric Properties

Vocabulary

Complementary Trig. Property	For any acute angle... $\cos\theta = \sin(90-\theta)$ & $\sin\theta = \cos(90-\theta)$
Tangent Property	For any acute angle... $\tan\theta = \frac{\sin\theta}{\cos\theta}$
Pythagorean Identity Property	For any acute angle... $(\sin\theta)^2 + (\cos\theta)^2 = 1$

Practice

For #1-5, fill in the blank using the properties.

1. sin $\underline{90-30}$ = cos 30°
   $\boxed{60°}$

2. cos 27° = sin $\underline{90-27}$
   $\boxed{63°}$

3. tan45° = $\boxed{\frac{\sin 45°}{\cos 45°}}$

4. $(\sin 20°)^2 + (\cos \boxed{20}°)^2 = 1$

5. $\frac{\sin 50}{\cos 50} = \boxed{\tan 50°}$

For #6-8, use the properties to solve the following problems.

6. If cos$x$ = 0.52, find sin$x$.

$(\sin x)^2 + (\cos x)^2 = 1$

$(\sin x)^2 + (.52)^2 = 1$

$(\sin x)^2 + .2704 = 1$

$-.2704 \quad -.2704$

$\sqrt{(\sin x)^2} = \sqrt{.7296}$

$\boxed{\sin x = .854}$

7. If sin$x$ = 0.848 and cos$x$ = 0.530, find tan$x$.

$\tan x = \frac{\sin x}{\cos x}$

$\tan x = \frac{.848}{.530}$

$\boxed{\tan x = 1.6}$

8. Suppose cos$x$ = 0.438 and tan$x$ = 2.050. Find sin$x$.

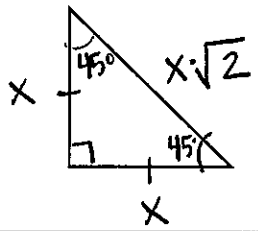
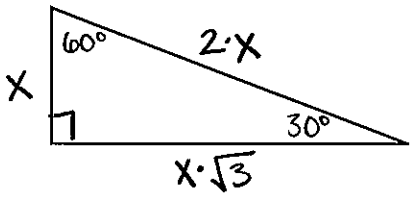
$\tan x = \frac{\sin x}{\cos x}$

$.438 \cdot 2.050 = \frac{\sin x}{\cancel{.438}} \cdot \cancel{.438}$

$\boxed{.8979 = \sin x}$

Lesson 10-3 Day #2: Special Right Triangles w/ Exact Values

There are two types of special right triangles you learned about last year in Geometry...

Special Right Triangles	
Isosceles Right Triangle	
30-60-90	

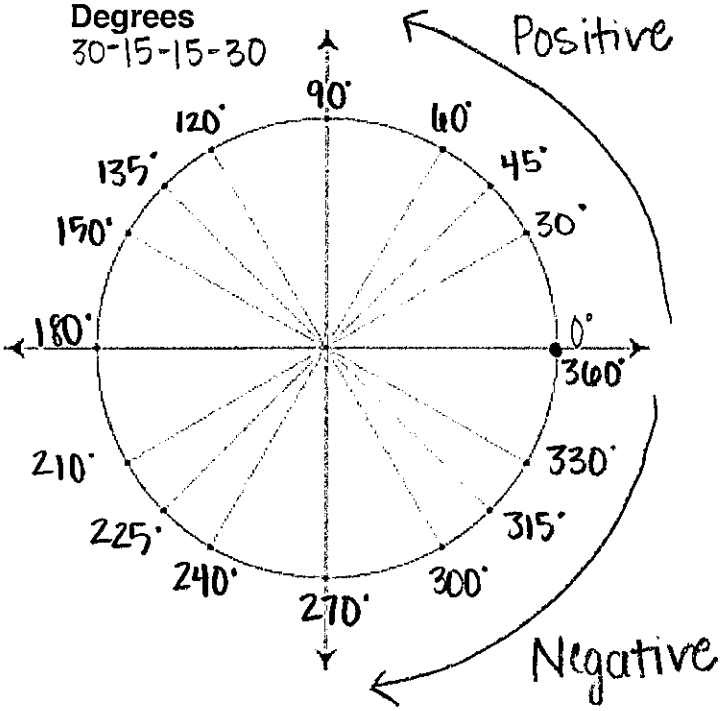
Using those side lengths & SOH-CAH-TOA, fill in the table below with EXACT VALUES ("exact" means <u>no</u> <u>decimals</u>!). Be sure to rationalize the denominator!!!

	45°	30°	60°
Sine SOH	$\sin 45 = \frac{\cancel{x}}{\cancel{x}\sqrt{2}}$ $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$	$\sin 30° = \frac{\cancel{x}}{2\cancel{x}}$ $\boxed{\frac{1}{2}}$	$\sin 60 = \frac{\cancel{x}\sqrt{3}}{2\cancel{x}}$ $= \boxed{\frac{\sqrt{3}}{2}}$
Cosine CAH	$\cos 45° = \frac{\cancel{x}}{\cancel{x}\sqrt{2}}$ $\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$	$\cos 30 = \frac{\cancel{x}\sqrt{3}}{2\cancel{x}}$ $= \boxed{\frac{\sqrt{3}}{2}}$	$\cos 60 = \frac{\cancel{x}}{2\cancel{x}}$ $= \boxed{\frac{1}{2}}$
Tangent TOA	$\tan 45° = \frac{\cancel{x}}{\cancel{x}}$ $= \boxed{1}$	$\tan 30° = \frac{\cancel{x}}{\cancel{x}\sqrt{3}}$ $\frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$	$\tan 60 = \frac{\cancel{x}\sqrt{3}}{\cancel{x}}$ $= \boxed{\sqrt{3}}$

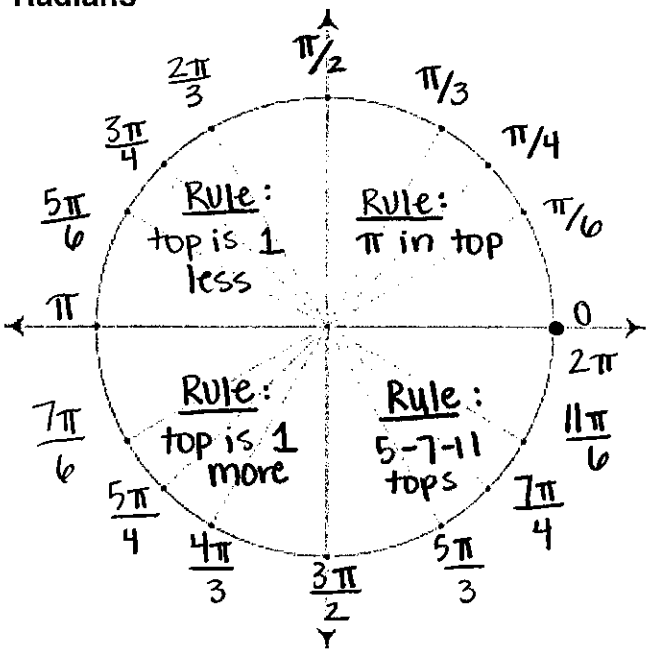
*****NOTE: You are responsible for MEMORIZING the nine values in the table above. Look for patterns & similarities to help!**

Lesson 10-4/10-5: Unit Circle

Degrees
30-15-15-30

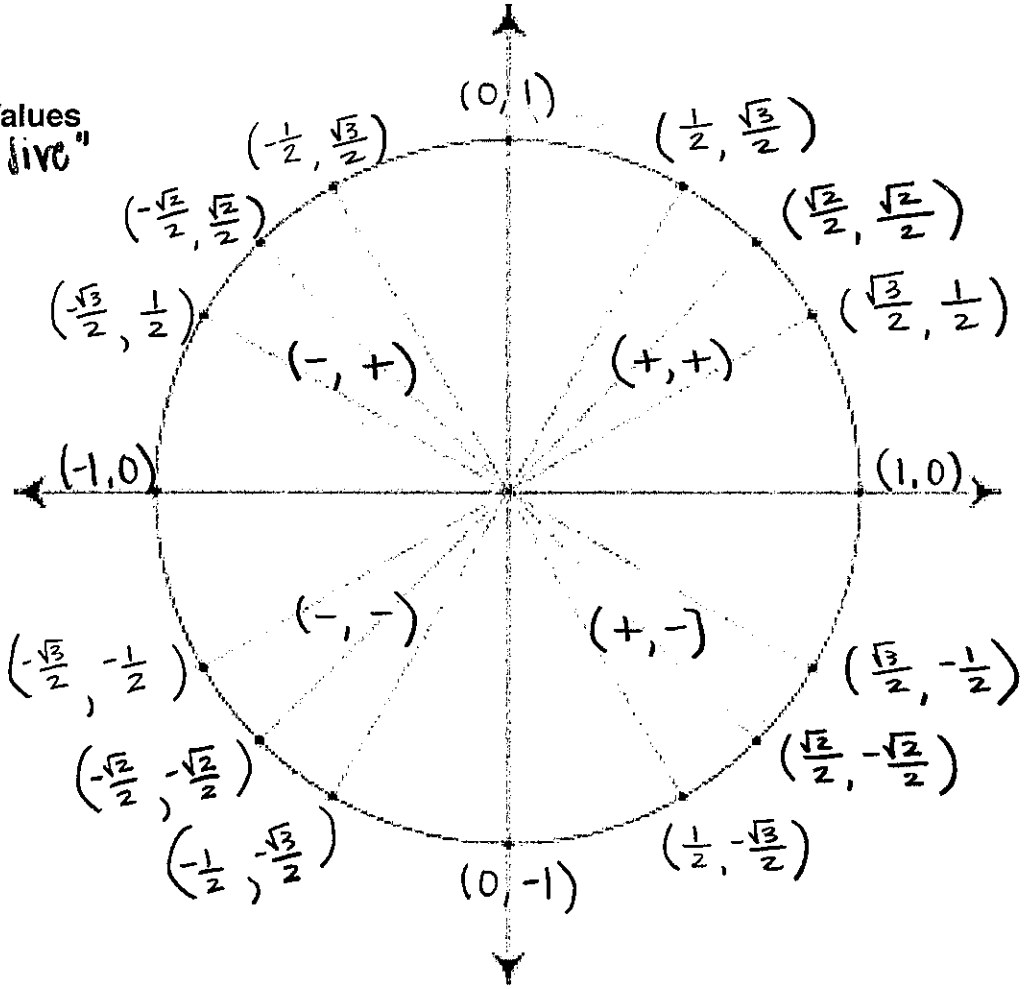


Radians



Exact Values
"Hand Live"

(cos, sin)



Practice

Use your unit circle to give the **exact value**. DO NOT USE A CALCULATOR!

1. cos0°

   1

2. sin120°

   $\frac{\sqrt{3}}{2}$

3. cos(-45°)

   $\frac{\sqrt{2}}{2}$

4. sin420°

   $\frac{\sqrt{3}}{2}$

5. cos(-315°)

   $\frac{\sqrt{2}}{2}$

6. sin180°

   0

7. $\cos\frac{3\pi}{4}$

   $-\frac{\sqrt{2}}{2}$

8. $\sin\frac{11\pi}{6}$

   $-\frac{1}{2}$

9. $\cos\frac{5\pi}{6}$

   $-\frac{\sqrt{3}}{2}$

10. $\sin\pi$

    0

11. $\cos\frac{\pi}{2}$

    0

12. $\cos 2\pi$

    1

Lesson: Law of Sines

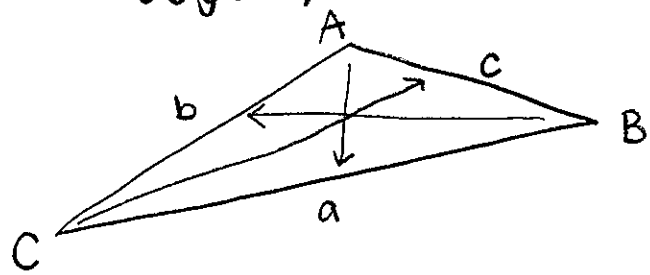
Vocabulary

no SSS, no SAS

Law of Sines: in a non-right △, $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

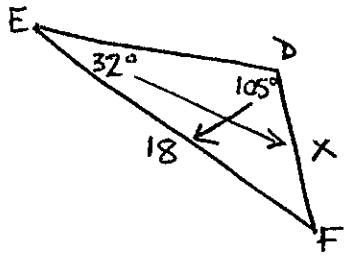
(A, B, & C are angles, a, b, & c are the sides across)

Example



Practice

1. Find the length of side *DF*.

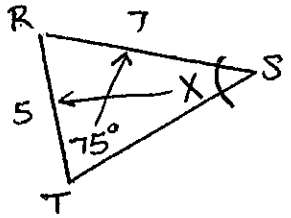


$$\frac{\sin 32°}{x} = \frac{\sin 105°}{18}$$

cross multiply & ÷ in calc!

$$\boxed{x = 9.88}$$

2. Find the measure of ∠S.



$$\frac{\sin x}{5} = \frac{\sin 75}{7}$$

cross mult. & ÷ in calc...

since it's an angle do inverse @ end!

$$\boxed{x = 43.6°}$$

Lesson: Law of Cosines

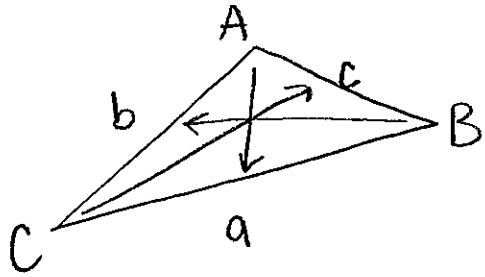
Vocabulary

SSS or SAS

Law of Cosines: in a non-right Δ, $a^2 = b^2 + c^2 - 2bc \cdot \cos A$

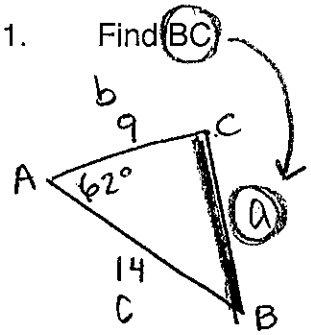
or $b^2 = a^2 + c^2 - 2ac \cdot \cos B$, or $c^2 = a^2 + b^2 - 2ab \cdot \cos C$

Example



Practice

1. Find BC



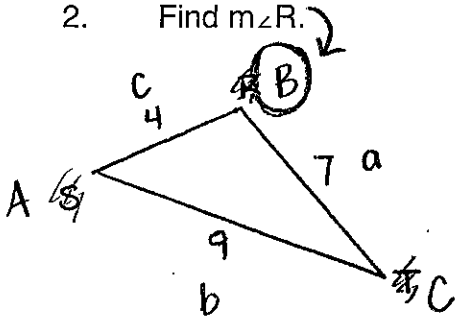
$a^2 = b^2 + c^2 - 2bc \cdot \cos A$

$a^2 = \underbrace{9^2 + 14^2 - 2 \cdot 9 \cdot 14 \cdot \cos 62°}_{\text{calc}}$

$a^2 = 158.7$

$\boxed{a = 12.6}$

2. Find m∠R.



$b^2 = a^2 + c^2 - 2ac \cdot \cos B$

$9^2 = \underbrace{7^2 + 4^2}_{\text{calc}} - \underbrace{2 \cdot 7 \cdot 4}_{\text{calc}} \cdot \cos B$

$81 = 65 - 56 \cdot \cos B$

$-65 \quad -65$

$\frac{16}{-56} = \frac{-56 \cos B}{-56}$

$-.2857 = \cos B$

$\cos^{-1}(-.2857) = B$

$\boxed{106.6° = B}$